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CCLIII.

(Vol. XII.—January, February, March, 1883.)

DESCRIPTION OF SOME EXPERIMENTS ON THE FLOW OF WATER MADE DURING THE CONSTRUCTION OF WORKS FOR CONVEYING THE WATER OF SUDBURY RIVER TO BOSTON.

By A. FTELEY, M. Am. Soc. C. E., and F. P. STEARNS, M. Am. Soc. C. E.

PRESENTED AT THE ANNUAL CONVENTION, MAY 19TH, 1882.

AND FOR WHICH THE NORMAN MEDAL WAS AWARDED JANUARY 17TH, 1883.

The experiments were made by the direction of Mr. J. P. Davis, M. Am. Soc. C. E., then City Engineer, for the ultimate purpose of determining with accuracy the flowing capacity of the conduit which conveys the water of the new source to the city, and of ascertaining experimentally the correctness of gaugings made for a series of years, with apparatus of various forms, of the flow of Sudbury River, and of other streams connected with the water supply of Boston. These researches led incidentally to some other observations which were not originally contemplated.

This paper relates chiefly to experiments upon the flow of water over the weirs which were used as measuring apparatus. The experiments and methods used are described at considerable length in order to give the reader a correct idea of the various circumstances under which the results were obtained; but many small details are necessarily omitted which were found essential to the accuracy of the experiments.

The endeavor has been made to represent the results by such formulæ as would require the least labor in their practical application, and, to this end, when it was found necessary to choose between a complicated expression and a more simple one with a variable co-efficient, the latter has been preferred; and a table of co-efficients has been annexed. In some cases simple formulæ have been given, which, although only approximate, are sufficiently correct for general practice; but in all cases the reader can, from the description of the methods used and of the results obtained, form an idea of the degree of accuracy of the formulæ given.

The Sudbury River works consist of a series of storage reservoirs constructed in the valley of the stream; the water drawn from them is regulated by gates, and flows into a natural pond (Farm Pond) 165 acres in area, from which it is conveyed, through a gate-house and a conduit, to the distributing reservoirs sixteen miles distant. The water used for the experiments was drawn from Farm Pond; the weirs and other gauging apparatus were located either in the gate-house or in the adjacent portion of the conduit.

The principal feature of the apparatus can be understood from the Plates I., II. and III., which will be referred to and explained in detail as we proceed in the description of the gaugings.

Plate I. represents the various apparatus used in the spring of 1877.

They were used for determining the discharge over a sharp-crested weir, with depth of water varying between 0.08 and 0.82 foot, the effect of various changes of forms of the crest of the weir, the discharge over a submerged weir, and also for determining the flowing capacity of the conduit.

Plate II. represents the apparatus used in the spring of 1878 to determine the influence of the velocity of approach on the flow of water over the weir, and the effect of end contraction.

Plate III. shows the apparatus erected in the gate-house, and used

in the winter of 1879–80 to measure the discharge with depths from 0.46 to 1.60 over the weir, to determine the flowing capacity of the conduit for large volumes, and to test the accuracy of current-meter measurements.

Flow of Water over Weirs.—General Remarks.

These experiments having been attempted originally with the intention of measuring the flow in the conduit by the means of a weir, the first step taken was to erect one in such conditions as should be warranted by previous practice, and to verify the accuracy of the methods used by actual observations.

The flow of water over a weir is the result of the action of gravity modified by various secondary causes, such as the velocity of approach, the resistance of the air, the temperature, the adhesion of the water to the edge of the weir, etc. To these causes which tend to modify the flow, and which cannot be entirely eliminated, others may incidentally be added, such as the end contractions, the form of the crest, etc., etc.

In practice we are, for lack of data, compelled to include in the main formula used for computing the flow the influence of such secondary modifying causes as the viscosity of the water, the resistance of the air, etc.; but the effect of all modifying causes which are not so included must be computed separately.

To obtain the value of the co-efficient in the main formula it is essential to measure the quantity of water passing the weir, and it is desirable to experiment on a weir where the flow is modified as little as possible by secondary causes.

To this end the velocity of approach should be made as small as practicable by deepening the channel above the weir, and end contractions should be avoided.

The co-efficient to be used in the main formula being obtained, the value of the secondary causes which tend to modify the flow may be determined by experiments, during which the modifying causes are introduced in succession, while the volume flowing over the weir remains constant.

This method has been resorted to in many of these experiments. It has the merit of eliminating some errors, of furnishing results rapidly, and, when the volume flowing remains very nearly constant, of furnishing results which cannot be surpassed for accuracy.

A constant flow was easily obtained from Farm Pond. Its natural outlet was closed by a dam, and its water shed was so small, when compared with its area, that the level of the pond, when no water was being used, scarcely varied from day to day, except during a rain. In 1877 and 1878, when the experiments requiring a constant flow of water were made, the largest draught caused a lowering of the surface of about 0.015 foot per hour.

Water was drawn from the pond into the weir basin through orifices on which the head was three feet or more; consequently a variation of 0.01 foot in the level of the pond would affect the head on the orifices but $\frac{1}{300}$, and since the volume flowing through an orifice varies as the square root of the head, the variation in volume would be only about one-half as much, or $\frac{1}{600}$.

This represents nearly the extreme variation due to change in the level of the pond. During the experiments of 1877 the pond was covered with ice. In 1878 there was no ice on the pond, and some fluctuations in the channel of the weir were caused by the waves.

To illustrate the method of experimenting, let us suppose that it is to be applied to the study of the effect of a wide crest on the flow.

The weir experimented on being originally constructed in its most simple form with a sharp crest, a constant quantity of water is made to pass over it; the depth of water on this weir is measured a sufficient number of times to ascertain that the flow is practically constant; if, then, by a suitable arrangement, a wide crest is quickly substituted for the former, without otherwise changing the conditions of the apparatus, the variation of the depth over the weir, as ascertained by a number of measurements sufficient to give a reliable result, will indicate the effect of the change on the flow.

The advantage of this method over the usual one of measuring the water in a basin is due to the fact that it eliminates entirely the errors due to the measurement of the basin and to the observation of the duration of the experiments, and, to a great extent, the inaccuracies connected with the beginning and end of each experiment, with the measurements of length of weir, comparison of gauges with the weir, leakage at weir, irregular velocity of approach, etc., as the circumstances of the flow are so nearly alike before and after the weir is modified.

EXPERIMENTS ON "VELOCITY OF APPROACH."

During some experiments made in 1877 (apparatus shown on Plate I.), as the flow over the weir was being gauged by the methods indicated by Mr. J. B. Francis, and measured afterwards in the basin below, it was incidentally observed that the increase of flow due to the velocity of approach appeared to be greater than was indicated by his formula. A study of the observations and of these experiments of Mr. Francis* and of Boileau† indicated that, in order to calculate the effect of velocity of approach, the theoretical head due to that velocity must be multiplied by 1.8, 1.5 and 1.8 respectively. That Mr. Francis does not think his formula strictly applicable to all cases may be seen from the following quotations from his work:‡

"When the sill (of the weir) is at a height above the bottom of the reservoirs, not less than twice the height of the water above the sill, and the sides (of the reservoir) are removed (from the side of the weir) a distance at least equal to the height above the sill, a correction free from serious error can usually be made for the effect of the velocity of the water approaching the weir."

"The experiments indicate that the method is not strictly accurate, as might be anticipated, omitting, as it does, all consideration of the effect produced by this velocity in modifying the contraction."

"It is well known that such an effect is produced, but it is of such a complicated nature that the investigations hitherto undertaken have thrown but little light upon it."

These quotations illustrate well the reasons which led to a series of investigations on the subject; these reasons being :—to find a correction when the cross-section of the channel of approach is not within the limits to which Mr. Francis' formula applies, and to obtain more accurate corrections, based on more numerous experiments made under a variety of conditions as regards velocities, depths of channel, etc.

It is, however, almost impossible to cover all cases that may occur in practice.

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* Lowell Hydraulic Experiments, by James B. Francis, M. Am. Soc. C. E., etc., N. Y. 3d edition. 1871.

† P. Boileau Traité de la mesure des eaux courantes. Paris, 1854.

‡ Lowell Hydraulic Experiments, pp. 72 and 120.

The mean velocity of approach is the quotient obtained by dividing the volume passing the weir by the area of the water section in the channel of approach.

With a weir having end contractions, the water section in the channel may assume many different forms with the same area, and consequently with the same mean velocity. As the effect of the velocity of approach may change with the form of the section, it would be necessary, in order to cover all cases, to make a large number of experiments.

This objection is removed by the use of a weir without end contractions, as in this case the mean velocity, for any depth on the weir, is dependent upon the depth of the channel, but not upon its width; the experiments will show what influence, if any, the proximity of the bottom of the channel may have upon the bottom contraction, and this effect will be constant for all weirs of similar construction, regardless of their length.

For the reasons just given, the experiments were made chiefly upon a weir without end contractions, although some of them were made upon weirs with end contractions. It will be seen, also, that some observations were made to determine the best methods for measuring the head on the weir, and to investigate other points relating to the subject of velocity of approach.

The apparatus used almost exclusively for this series of observations is shown on Plate II.

The water used during the experiments was drawn from Farm Pond and flowed thence into the gate-house, and in the portion of the adjacent conduit which contained the measuring weir.

Fig. 1 is a longitudinal section. Fig. 2 a plan in which the upper part of the conduit is supposed to be removed. Fig. 3 is an up-stream elevation of the weir, showing also a cross-section of the conduit. *A*, *A* are stop-planks placed in one of the grooves of the stone work of the gate-house; *B*, *B* are openings in the stop-planks, and *C*, *C* are gates which, together with the stop-planks, controlled the entrance of the water from Farm Pond. The gates were movable horizontally to admit the desired quantity of water, and could be firmly clamped in any position, thus securing a constant area of orifice.

The water on the up-stream side of the stop-planks communicated with the water in Farm Pond through large openings in three masonry walls (shown at *A*, *D*, *G*, Fig. 1, Plate III.). Owing to the depth of these

openings below the water surface and to the water spaces between the walls, large waves on the pond caused but a slight rise and fall of the water at the stop-planks. The openings *D*, *G*, in two of the walls, were fitted with iron gates, but during the experiment they were kept wide open. The water, after passing the orifices in the stop-planks, fell into two wooden boxes (*D*, *D*, Plate II.), where it spread in all directions, and finally found egress through two long, narrow openings *E*, *E*, near the stop-planks. The surface of the water below the boxes showed little agitation, but it was further calmed by three floating planks *F*, *F*, *F*. *G* is a screen at the entrance of the conduit. It was made of boards, 6 inches wide, placed 4 inches apart, and, after a few experiments, laths were nailed horizontally, about 2 inches apart, on the down-stream side for the purpose of increasing the loss of head at this point.

The weir *S* was placed 33 feet from the entrance to the conduit; its channel of approach was 25 feet in length and 5 feet in width; the sides of the channel formed the end of the weir and thus prevented end contractions, they also extended a short distance on the down-stream side of the weir as represented at *T*, to prevent the lateral expansion of the sheet after passing the weir.

The bottom of the channel of approach was a movable platform made in three sections, *L*, *K* and *I*. These sections were connected by hinges and their lengths were respectively 10, 8 and 5 feet. The two sections nearest the weir, *L* and *K*, could be maintained horizontally at five different depths, varying from 0.50 to 3.56 feet below the crest of the weir. The section farthest from the weir, *I*, was level with the other sections when the platform was at its lowest position, as shown by the full lines in Fig. 1, but when sections *L* and *K* were maintained at any higher level, section *I* was inclined. The dotted lines show the position of the platform when at a higher level.

The method of raising and lowering the platform may be understood from the following description and an examination of Fig. 1. Let us first suppose that it is to be raised from the lowest position, as represented by the full lines, to the position shown by the dotted lines. Stops were first placed on the sides of the channel, as shown by the dotted lines at *N*, *N*, *N*, to prevent the platform from raising above the desired height. By means of the levers *H*, *H*, the platform was then drawn from its fastenings in the notches *P*, *P*, *P*.

The flap *I* was next raised to admit the water under the platform,

when, on account of its buoyancy, the latter rose until prevented by the stops. The levers were again used to force the platform towards the weir, and to secure it in the notches at the desired height. During the experiments the flap was fastened down and the stops were removed.

To lower the platform without stopping the flow of water, it was necessary to draw it from its fastenings by means of the levers, to open the gate at the weir, and to keep the flap closed. By these means the water was drawn from the space beneath the platform, causing it to lower. When section *K* had reached the bottom, stops of the proper length were put on to keep it in that position, and the end of section *L*, which struck against the open gate at the weir, was forced down after the gate was closed. By again using the levers the platform was forced towards the weir, and thus secured in its original position. *M* is a screen made in three parts to allow portions of it to be removed as the platform was raised. The openings in this screen occupied $\frac{4}{7}$ of its whole area, the slats were 0.026 ft. thick. Between the screen and the weir the channel was uniform in section, and without any obstruction to the flow, except that caused by the hinges on the platform and by a few pieces nailed on the sides to prevent the platform from rising above the highest notch.

The sides and bottom of the channel were made of planed boards, tongued and grooved. Portions of the side, near and at the ends of the weir, were carefully made both for the purpose of having true planes to limit the length of the weir, and of having smooth boards to diminish the friction at this point where the velocity of the water is greatest.

Leakage into the channel from the bottom and sides, except at points distant from the weir, was prevented as far as practicable. The boarding itself was practically water-tight. The movable bottom was about $\frac{1}{16}$ of an inch narrower than the channel.

Water was prevented from entering the opening in the side of the channel at *Q* (Fig. 1), by a second row of boarding shown at *R* (Fig. 2).

As a further precaution against leakage, the water in the spaces behind the sides and under the bottom of the channel did not have communication with the water above the screens, but communicated freely with the water below the screens through the openings *O*, *O*. This arrangement prevented any considerable pressure on the partitions. The weir was of hard pine, the edge presented to the current being sharp; the down-stream side was chamfered so that the water in no case touched any part of the weir after passing the up-stream edge. The depth on the weir was

measured in two different manners. Two hook gauges of the same patterns were placed on opposite sides of a post in the centre of the conduit 6 feet below the weir, as represented at *U*.

For convenience of reference the gauges have been designated as "No. 1" and "No. 2."

The hook gauge pails communicated with the basin above the weir by rubber pipes.

The rubber pipe leading to the pail at gauge "No. 1" was connected at the other end with a gas pipe *V*, of one inch inside diameter, placed horizontally and transversely at the bottom of the channel. The gas pipe was stopped at both ends and perforated at the top with eight ¼-inch holes, 0.63 feet apart. The pipe rested upon the movable platform, and the holes were 0.11 feet from the face of the weir, and at an equal distance above the platform.

The rubber pipe leading to the pail at gauge "No. 2," was connected at the other end with an opening in a piece of plate glass, represented at *W*. The glass was set flush with the side of the channel, and as far as could be determined was perfectly straight and smooth. The orifice in the glass was 0.04 feet in diameter, and was originally 6 feet from and 0.414 feet below the level of the crest of the weir; it was afterwards moved 0.02 feet higher.

The hook gauges were of a well-known pattern. A similar one was used by Gen. T. G. Ellis, at Holyoke, and is illustrated in the Transactions of the Society for February, 1876. Another illustration of the same gauge is given in Mr. J. T. Fanning's "Water Supply Engineering," page 297.

For the purpose of studying the effect of velocity of approach over a weir, with end contractions, pieces were provided to shorten the weir and to cause either one or two end contractions.

The experiments were made in series. The routine of each was as follows:

The gates *C*, *C*, admitting the water, were opened to the desired width and clamped. The almost constant head upon these gates, together with the constant area of their openings, caused a very nearly constant volume to flow over the weir. The measurements were first taken when the depth of water in the channel was greatest, and afterwards for each of the lesser depths at which the bottom of the channel could be maintained. At the end of a series, the first measurement was repeated to

detect any change which might have occurred in the volume of water flowing. Each recorded depth on the weir was a mean of several (generally eight or ten) measurements.

In some of the experiments first made, owing to defects in the apparatus, the movable bottom could not be put back to its lowest position to allow the first measurement to be repeated.

The effect of the velocity of approach upon the discharge over a weir has been formulated in various ways.

The most usual way, and apparently the most rational (since the velocity of approach affects directly the depth on the weir), consists in correcting the observed depth on the weir before using the formula of discharge.

As we know that the effect of velocity of approach on the depth over the weir varies approximately with the head due to that velocity, it has been endeavored to find by what co-efficient that head is to be multiplied, under different conditions, to give correct results.

Therefore, the formula used to express the correction to be made for the effect of the velocity of approach is of the following form :

$$C = ch, \text{ in which—}$$

C is the quantity to be added to the observed depth on the weir.

c is the co-efficient modifying the value of h. h is the head due to the velocity of approach, found in each case by the well-known formula $h = \frac{v^2}{2g}$, in which v is the mean velocity of approach, obtained by dividing the volume of discharge by the area of the water section of the channel.

The experiments referring to this subject were made, as has been said before, with various conditions of the weir, each of which will be considered separately. They are as follows :

1st. Weir without end contractions, the head being taken 6 feet above the weir (gauge No. 2).

2d. Weir with end contraction (same gauge).

3d. Weir without end contractions, the head being taken near the weir at the bottom of the channel (gauge No. 1).

4th. Weir with end contractions (same gauge).

1st.—Weir without End Contractions. Head taken 6 feet above Weir (Gauge No. 2).

The co-efficient found, though not far from constant, varied somewhat with the depth of the channel and the depth on the weir ; but we failed to discover any variation in the co-efficient due to a change in the mean velocity of approach.

Table I. gives the values of the co-efficients found for different depths of channel and for different depths of water on the weir, and embodies the results of all the experiments. The total effect of the velocity of approach, as calculated by the formula, is also shown by the small figures in the table.

From an examination of these figures, it will readily be seen that in some cases the total effect of the velocity of approach is so small that it can scarcely be measured, while in others it is quite a large quantity. As a consequence the co-efficients based upon the measurements may be, in the former cases, quite indefinite ; in the latter they will scarcely vary a unit in the last figure.

It may also be seen that the co-efficient 1.5 would be sufficiently accurate for a majority of cases in practice ; the general formula would then become for this form of weir

$$C = 1.5\ h.$$

To illustrate more fully the results obtained, Table II. has been prepared to show the detail of the experiments, and a comparison of each with the formula adopted.

Columns 1 to 4 are sufficiently explained by their headings.

Column 5 shows the supposed depth on the weir as it would be, if there were no velocity of approach.

TABLE I.

Table, showing the value of c in the formula $C = ch$, corresponding to various depths on the weir, and to various depths of the channel of approach. Weir without end contractions. Heads measured by gauge No. 2, six feet from weir.

1	Depths of Channel of Approach below Crest of Weir.			
Depths on Weir as measured.	2	3	4	5
	2.60	1.70	1.00	0.50
		0.0007	0.0019	0.0054
0.20	0.0003	1.66	1.87	1.70
	1.51	0.0020	0.0054	0.0130
0.30	0.0009	1.65	1 83	1.53
	1 50	0.0043	0.0111	0.0258
0.40	0.0019	1.63	1 79	1.53
	1.49	0.0076	0.0193	0.0425
0.50	0.0034	1.62	1.75	1.53
	1 48	0.0121	0.0291	0.0635
0.60	0.0056	1.60	1.71	1.52
	1.47	0.0180	0.0412	0.0889
0 70	0.0083	1.59	1.68	1.51
	1.46	0.0243	0.0550	0.1188
0.80	0.0117	1.57	1 65	1.50
	1.45	0.0323	0.0714	0.153
0.90	0.0156	1.56	1.63	1.49
	1.44	0.0411	0.090	0.192
1.00	0.0202	1.54	1.61	1.48
	1 43	0.0513	0.111	
1 10	0.0254	1.53	1 59	
	1.42	0.0623	0.132	
1.20	0.0314	1 51	1.57	
	1 41	0.074	0.156	
1.30	0.0380	1.49	1.55	
	1.40	0.087	0.183	
1.40	0.0453	1.48	1.54	
	1,39	0.101	0.209	
1.50	0.053	1.46	1.52	
	1 38	0.115	0.240	
1.60	0.061	1.44	1.51	
	1.37	0.131	0.269	
1.70	0.070	1.43	1.49	
	1.36	0.146		
	0.080	1.41		
	1.35	0.164		
1.90	0.090	1.40		
	1.34	0.180		
2.00	0.100	1.38		
	1.33			

The co-efficients in this column may be used for greater depths of channel.

Note.—The co-efficients above the dotted lines are within the limits of the experiments.

As, however, the velocity of approach could not be entirely eliminated in any case, that result was obtained by correcting the observed depth on the weir in the experiments in which the velocity of approach was least. This correction, which was approximate at the first, was afterwards made with more accuracy from the results of the other experiments, which furnished the basis for successive approximations.

Column 6 was obtained by dividing the discharge over the weir by the section of the channel of approach.

Column 8 shows the effect of the velocity of approach, and, for each series, is obtained by subtracting the depths in column 4 from the depths in column 5.

The first line in each series is left blank, as nothing could be shown but the quantity previously calculated to obtain the corrected depth given in column 5.

Column 10 is explained by the heading.

The last experiment in several series (seven experiments in 94) is marked with an asterisk ; they are more or less defective, owing to the fact that when the platform was at its highest point, the orifice of the pipe of gauge No. 2 was too near the bottom of the channel ; owing to this and to other defects in the apparatus, the observed depth on the weir was affected by the pressure of the water in the space beneath the platform.

These defects were remedied on March 15th, but the same trouble occurred again during two experiments on March 19th.

Although some of the defective experiments were but slightly erroneous, it was thought best, in determining the co-efficient of the formula, to exclude them.

The maximum difference in column 10, disregarding the quantities marked with an asterisk, is in experiment 58, +0.0021 ft.

In no case does the discrepancy correspond to a difference in volume exceeding one-half of one per cent. The mean difference, disregarding signs, is 0.0005, corresponding to a mean difference in volume of about one-eighth of one per cent. It is thought that smaller differences might have been obtained had it not been for the prevalence of strong winds on the pond during many of the experiments, causing the quantity of water, which passed the weir, to vary slightly from time to time.

The construction of the apparatus had been hastened in hopes that the ice, then covering the pond, would remain during the experiments,

TABLE II.

Experiments on Velocity of Approach.

Weir without End Contraction. Heads taken near the surface six feet up-stream from the weir. Temperature of the water about 40°.

Note.—Experiments marked with an asterisk are defective.

1 Number of the Series and of the Experiment.	2 Date of the Experiment. 1878.	3 Depth of Channel of Approach below Crest of Weir.	4 Depth on the Weir measured near the Surface 6 feet up-stream from the Weir. Gauge No. 2.	5 Depth on the Weir corrected for Velocity of Approach.	6 Mean Velocity of Approach in feet per second.	7 Theoretical Head due to Mean Velocity of Approach.	8 Effect of Velocity of Approach as Observed in each Experiment.	9 Effect of Velocity of Approach as Calculated by the Formula $C = c\,h$, c having the value given in Table I.	10 Difference of Effect as obtained by Experiment from Effect as obtained by Formula.
Series I.		....	.....	0.1932	.....	.....	.....	.....	.....
Exp. 1	March 16th	3.56	0.1930	.....	0.077	0.0001	.....	0.0002	.....
" 2	"	1.70	0.1924	.....	0.153	0.0004	0.0008	0.0006	+0.0002
" 3	"	1.00	0.1913	.....	0.243	0.0009	0.0019	0.0017	+0.0002
" 4	"	0.50	0.1884	.....	0 420	0.0027	0.0048	0 0047	+0.0001
Series II.		....	.....	0.2688	.....	.....	.....	.....	.....
Exp. 5	March 16th	3.56	0.2685	.....	0.123	0.0002	.....	0.0003	.....
" 6	"	2.60	0.2685	.....	0.164	0.0004	0.0003	0.0006	—0.0003
" 7	"	1.70	0.2676	.....	0.239	0.0009	0.0012	0.0015	—0.0003
" 8	"	1.00	0.2649	.....	0.372	0.0022	0.0039	0.0040	—0.0001
" 9	"	0.50	0.2595	.....	0.620	0.0060	0.0093	0.0095	—0.0002

Series III				0.3368					
Exp. 10	March 12th	3.56	0.3361		0.168	0.0004		0 0007	
" 11	"	2.60	0.3358		0.224	0.0008	0.0010	0.0012	−0.0002
" 12	"	1.70	0.3341		0 323	0.0016	0.0027	0.0026	+0.0001
" 13	"	1.00	0.3308		0.494	0.0038	0.0060	0.0069	−0.0009
* " 14	"	0.50	0.3188		0.802	0.0100	0.0180	0.0153	* +0.0027
Series IV				0.3578					
Exp. 15	March 16th	3.56	0.3570		0.183	0.0005		0.0008	
" 16	"	2.60	0.3561		0.244	0.0009	0 0017	0.0014	+0.0003
" 17	"	1.70	0.3549		0.350	0.0019	0.0029	0.0031	−0.0002
" 18	"	1.00	0.3495		0.533	0.0044	0.0083	0.0080	+0.0003
" 19	"	0.50	0.3404		0.856	0.0114	0.0174	0.0174	0.0000
Series V				0 4232					
Exp. 20	March 16th	3.56	0.4219		0.232	0.0008		0.0013	
" 21	"	2.60	0.4205		0.306	0.0015	0.0027	0.0022	+0.0005
" 22	"	1.70	0.4183		0.436	0.0030	0.0049	0.0048	+0.0001
" 23	"	1.00	0.4106		0.655	0 0067	0.0126	0.0119	+0.0007
" 24	"	0.50	0.3976		1.029	0.0165	0.0256	0.0252	+0.0004
Series VI				0.4276					
Exp. 25	March 15th	3.56	0.4263		0 235	0.0009		0.0015	
" 26	"	2.60	0.4257		0.310	0.0015	0.0019	0.0022	−0.0003
" 27	"	1.70	0.4230		0.442	0.0030	0.0046	0.0049	−0.0003
" 28	"	1.00	0.4154		0.663	0.0068	0.0122	0.0122	0.0000
* " 29	"	0.50	0.3997		1.043	0.0169	0.0279	0.0259	* +0 0020

TABLE II.—(*Continued.*)

1	2	3	4	5	6	7	8	9	10
Number of the Series and of the Experiment.	Date of the Experiment. 1878.	Depth of Channel of Approach below Crest of Weir.	Depth on the Weir measured near the Surface 6 feet up-stream from the Weir. Gauge No. 2.	Depth on the Weir corrected for Velocity of Approach.	Mean Velocity of Approach in feet per second.	Theoretical Head due to Mean Velocity of Approach.	Effect of Velocity of Approach as Observed in each Experiment.	Effect of Velocity of Approach as Calculated by the Formula $C = c\,h$, c having the value given in Table I.	Difference of Effect as obtained by Experiment from Effect as obtained by Formula.
Series VII.	.....	....	.....	0.4952	....	.....	.....	.....	.....
Exp. 30	March 16th	3.56	0.4933	.....	0.288	0.0013	.....	0.0019	.....
" 31	"	2.60	0.4923	.....	0.377	0.0022	0.0029	0.0033	−0.0004
" 32	"	1.70	0.4886	.....	0.533	0.0044	0.0066	0.0072	−0.0006
" 33	"	1.00	0.4784	.....	0.790	0.0097	0.0168	0.0171	−0.0003
" 34	"	0.50	0.4597	.....	1.217	0.0230	0.0355	0.0352	+0.0003
Series VIII.	.....	....	.....	0.5170	....	.....	.....	.....	.....
Exp. 35	March 12th	3.56	0.5148	.....	0.306	0.0015	.....	0.0022	.....
" 36	"	2.60	0.5132	.....	0.400	0.0025	0.0038	0.0037	+0.0001
" 37	"	1.70	0.5088	.....	0.564	0.0050	0.0082	0.0081	+0.0001
" 38	"	1.00	0.4965	.....	0.833	0.0108	0.0205	0.0189	+0.0016
" 39	"	0.50	0.4706	.....	1.283	0.0256	0.0464	0.0392	+0.0072

Series IX				0.5647					
Exp. 40	March 20th	3.56	0.5620		0.345	0.0018		0.0027	
" 41	"	2.60	0.5596		0.450	0.0031	0.0051	0.0046	+0.0005
" 42	"	1.70	0.5550		0.630	0.0062	0.0097	0.0099	—0.0002
" 43	"	1.00	0.5409		0.922	0.0132	0.0238	0.0229	+0.0009
" 44	"	* 0.50	0.5187		1.394	0.0302	0.0460	0.0462	—0.0002
Series X				0.6026					
Exp. 45	March 21st	3.56	0.5994		0.376	0.0022		0.0032	
" 46	"	0.50	0.5509		1.490	0.0345	0.0517	0.0527	—0.0010
Series XI				0.6177					
Exp. 47	March 19th	3.56	0.6143		0.389	0.0024		0.0034	
" 48	"	2.60	0.6120		0.506	0.0040	0.0057	0.0058	—0.0001
" 49	"	1.70	0.6059		0.705	0.0077	0.0118	0.0124	—0.0006
" 50	"	1.00	0.5904		1.022	0.0162	0.0273	0.0278	—0.0005
* " 51	"	0.50	0.5685		1.522	0.0360	0.0492	0.0548	* —0.0056
Series XII				0.6785					
Exp. 52	March 12th	3.56	0.6741		0.440	0.0030		0.0044	
" 53	"	2.60	0.6707		0.571	0.0051	0.0078	0.0074	+0.0004
" 54	"	1.70	0.6629		0.791	0.0097	0.0156	0.0155	+0.0001
" 55	"	1.00	0.6443		1.137	0.0201	0.0342	0.0341	+0.0001
* " 56	"	0.50	0.6179		1.673	0.0435	0.0606	0.0661	* —0.0055
Series XIII				0.7019					
Exp. 57	March 19th	3.56	0.6971		0.462	0.0033		0.0048	
" 58	"	2.60	0.6917		0.598	0.0056	0.0102	0.0081	+0.0021
" 59	"	1.70	0.6836		0.826	0.0106	0.0183	0.0169	+0.0014
" 60	"	1.00	0.6658		1.181	0.0217	0.0361	0.0367	—0.0006
* " 61	"	0.50	0.6364		1.732	0.0466	0.0655	0.0707	* —0.0052

TABLE II.—(*Continued.*)

1 Number of the Series and of the Experiment.	2 Date of the Experiment. 1878.	3 Depth of Channel of Approach below Crest of Weir.	4 Depth on the Weir measured near the Surface 6 feet upstream from the Weir. Gauge No. 2.	5 Depth on the Weir corrected for Velocity of Approach.	6 Mean Velocity of Approach in feet per second.	7 Theoretical Head due to Mean Velocity of Approach.	8 Effect of Velocity of Approach as Observed in each Experiment.	9 Effect of Velocity of Approach as Calculated by the Formula $C = c\,h$, c having the value given in Table I.	10 Difference of Effect as obtained by Experiment from Effect as obtained by Formula.
Series XIV	.....	....	.....	0.7205	.....	.....	.....	.....	.....
Exp. 62.....	March 15th...	3.56	0.7153	.....	0.478	0.0036	.....	0.0052	....
" 63.....	" ...	2.60	0.7117	.....	0.617	0.0059	0.0088	0.0086	+0.0002
" 64.....	" ...	1.70	0.7018	.....	0.851	0.0113	0.0187	0.0179	+0.0008
" 65.....	" ...	1.00	0.6806	.....	1.217	0.0230	0.0399	0.0388	+0.0011
* " 66.....	" ...	0.50	0.6412	.....	1.792	0.0499	0.0793	0.0757	* +0.0036
Series XV.....	.....	....	.....	0.7817	.....	.....	.....	.....	.....
Exp. 67.....	March 18th...	3.56	0.7753	.....	0.533	0.0044	...	0.0064	....
" 68.....	" ...	2.60	0.7718	.....	0.685	0.0073	0.0099	0.0106	−0.0007
" 69.....	" ...	1.70	0.7606	.....	0.938	0.0137	0.0211	0.0216	−0.0005
" 70.....	" ...	1.00	0.7376	.....	1.329	0.0275	0.0441	0.0458	−0.0017
" 71.....	" ...	* 0.50	0.6922	.....	1.937	0.0583	0.0895	0.0881	+0.0014
Series XVI.....	.....	....	.....	0.7925	.....	.....	.....	.....	.....
Exp. 72.....	March 13th...	3.56	0.7859	.....	0.543	0.0046	.....	0.0066	.....
" 73.....	" ...	1.70	0.7695	.....	0.955	0.0142	0.0230	0.0224	+0.0006

Series XVII					0.8400					
Exp. 74	March 20th	3.56	0.8323		0 586	0.0053		0.0077		
" 75	"	2 60	0.8278		0.751	0.0088	0.0122	0.0127	−0.0005	
" 76	"	1.70	0.8147		1.023	0.0163	0.0253	0.0255	−0.0002	
" 77	"	1.00	0.7875		1 440	0.0322	0.0525	0.0533	−0.0008	
" 78	"	0.50	0.7409		2.072	0.0667	0.0991	0.1005	−0.0014	
Series XVIII					0.8736					
Exp. 79	March 11th	3.56	0 8651		0.617	0.0059		0.0085		
" 80	"	2.60	0.8595		0.789	0.0097	0.0141	0 0140	+0.0001	
" 81	"	1.70	0.8467		1.071	0.0178	0.0269	0.0279	−0.0010	
Series XIX					0.8899					
Exp. 82	March 20th	3.56	0.8809		0.632	0.0062		0.0090		
" 83	"	2.60	0.8755		0 807	0.0101	0.0144	0.0146	−0.0002	
" 84	"	1.70	0.8593		1.096	0.0187	0.0306	0.0292	+0.0014	
" 85	"	1.00	0.8293		1.534	0.0366	0.0606	0.0601	+0.0005	
" 86	"	0.50	0.7765		2.199	0.0752	0.1134	0.1129	+0.0005	
Series XX					0.9339					
Exp. 87	March 14th	3.56	0.9238		0.673	0.0070		0.0101		
" 88	"	2.60	0.9181		0.858	0.0114	0.0158	0.0165	−0.0007	
" 89	"	1.70	0.9011		1.160	0.0209	0.0328	0.0326	+0 0002	
" 90	"	1.00	0.8668		1.616	0.0406	0.0671	0.0665	+0.0006	
Series XXI					0.9550					
Exp. 91	March 22d	3 56	0.9443		0.693	0.0075		0.0107		
" 92	"	1.70	0.9215		1.191	0.0220	0.0335	0.0343	−0.0008	
" 93	"	1.00	0.8854		1.656	0.0426	0.0696	0.0696	0.0000	
" 94	"	0.50	0.8266		2.352	0.0860	0.1284	0.1288	−0.0004	

but it disappeared the day the apparatus was finished. The maximum limits reached by the experiments may be seen by an examination of Series XXI.

The maximum depth on the weir was about 0.95 feet. The various circumstances of Exp. 94, representing extreme conditions, is given below:

Depth of channel below crest of weir	0.50	feet.
Depth on weir, corrected for velocity of approach	0.9550	"
Depth on weir, as observed	0.8266	"
Effect of velocity of approach upon depth on weir	0.1284	"
Mean velocity of approach	2.352	"
Ratio of depth on weir to depth of channel below crest $\frac{.8266}{.5000}$	=1.65	
Co-efficient of formula	1.50	

The velocity of approach reduced the depth on the weir from 0.9550 to 0.8266 feet, or about 13½ per cent.

If the volume had been calculated from the observed depth on the weir, without correction for velocity of approach, the error in volume would have been about 19½ per cent. The most noticeable feature of the experiment is that, although the ratio of depth on weir to depth of channel is so much larger than is generally considered allowable, yet the co-efficient does not show any marked variation on this account. This would seem to warrant the inference that the extension of Table I. beyond the limits of the experiments was not grossly in error, though it may be somewhat inaccurate.

2d.—Weir with End Contraction. Heads taken 6 feet from the Weir (Gauge No. 2).

End contraction was caused by placing vertical pieces at one or both ends of the weir, thus diminishing its length. As the weirs with end contraction were necessarily shorter, one end contraction only was produced in many of the experiments. This form allowed the use of the longest weir practicable, and at the same time represented, approximately, the conditions existing in one-half of a weir of twice the length, having two end contractions.

The variations in the velocity of approach were accomplished by changing the height of the movable bottom of the channel, as in the experiments previously described.

They are too few to determine definitely how the co-efficients vary, but they show plainly that the effect of velocity of approach is much greater when end contraction exists, than when it does not. They also give indications of the following laws :

1st. That the effect of velocity of approach is greater with two end contractions than with one.

2d. That it increases as the proportion of the width of the channel occupied by the weir decreases.

3d. That it (and consequently the co-efficient) varies with the depth of the channel, and with the depth on the weir in the same way, and to nearly the same extent, as when there is no end contraction. An exception to this rule occurs in the case of the smallest depth of channel; the co-efficients, in this case, increase with the depth on the weir when end contraction exists, and decrease when it does not.

From the results of the experiments shown on Table No. III. (pages 22–23) a general co-efficient has been deduced which may be used with weir having end contractions.

The co-efficient is 2.05, and the formula becomes :—

$$C = 2.05\ h.$$

A rather more accurate co-efficient may be obtained by adding 0.55 to the co-efficient established for a weir without end contraction, as given in Table I.

Much more extensive observations than those under consideration would be required to determine the co-efficients for all forms of weirs and of the channels leading to them ; and much is left to the judgment of the observer, because the co-efficients found are correct only for an apparatus similar to the one which was experimented upon.

When a long weir occupies nearly the whole width of the channel leading to it, the conditions are but little different from the case of a weir without end contraction. On the other hand, a short weir, occupying but a small portion of the width of the channel, would need a larger co-efficient than any given.

The experiments are given in Table No. III. (pages 22–23). The first eight columns are the same as the corresponding columns in Table No. II., and the others are sufficiently explained by the headings.

TABLE III.

Experiments on Velocity of Approach.

Weir with End Contraction. Heads taken near the surface 6 feet up-stream from the Weir.

1 Number of the Series and of the Experiment.	2 Date of the Experiment. 1878.	3 Depth of Channel of Approach below Crest of Weir.	4 Depth on the Weir measured near the Surface, 6 feet up-stream from the Weir. Gauge No. 2.	5 Depth on the Weir corrected for Velocity of Approach.	6 Mean Velocity of Approach in feet per second.	7 Theoretical Head due to Mean Velocity of Approach.	8 Effect of Velocity of Approach as Observed in each Experiment.	9 Value of c in Formula $C=ch$.	10 Excess of Co-efficient in column 9 over Co-efficient for corresponding Depth of Channel and Depth on Weir as given in Table I.	Remarks.
Series I.	....	....	.....	0.8744	.....	.....	.....	.....	.....	Two end contractions.
Exp. 1	March 21st.	3.56	0.8702	.....	0.354	0.0020	.....	*2.14	*0.70	Length of weir, 3 feet.
" 2	"	1.70	0.8612	.....	0.613	0.0058	0.0132	2.26	0.70	Width of channel, 5 feet.
" 3	"	1 00	0.8490	.....	0.849	0.0112	0.0254	2.27	0.63	
" 4	"	0.50	0.8310	.....	1.180	0.0216	0.0434	2.00	0.50	
Series II.	....	....	.....	0.5782	.....	.....	.....	.....	... .	One end contraction.
Exp. 5	March 22d.	3.56	0.5765	.....	0.230	0.0008	.....	*2.02	*0.55	Length of weir, 3.3 feet.
" 6	"	1.00	0.5647	.....	0.609	0.0057	0.0135	2.35	0.63	Width of channel, 5 feet.
" 7	"	0.50	0.5574	.....	0.901	0.0126	0.0208	1.65	0.13	
Series III.	....	....	.....	0.8102	.....	.....	.....	.....	.....	Conditions same as for Series II.
Exp. 8	March 21st.	3.56	0.8062	.....	0.360	0.0020	.....	*2.00	*0.55	
" 9	"	2.60	0.8036	.....	0 462	0.0033	0.0066	1.99	0.54	
" 10	"	1.70	0.7969	.....	0.629	0 0061	0.0133	2.16	0.59	
" 11	"	1.00	0.7850	.....	0.880	0.0120	0.0252	2.09	0.44	
" 12	"	0.50	0.7677	.....	1.239	0.0239	0.0425	1.78	0.28	

Series	IV.	.....	....	.....	0.9365	.....	.....	.....	.....	.....	Conditions same as for Series II. and III.
Exp.	13	March 22d.	3.56	0.9307	.....	0.434	0.0029	.....	*1.99	*0.55	
"	14	"	1.00	0.9024	.....	1.021	0.0162	0.0341	2.10	0.47	
"	15	"	0.50	0.8775	.....	1.410	0.0309	0.0590	1.91	0.42	
Series	V.	.....	....	.....	0.7086	.....	.....	.....	.....	.....	One end contraction.
Exp.	16	March 21st.	3.56	0.7048	.....	0.368	0.0021	.....	*1.81	*0.35	Length of weir, 4 feet.
"	17	"	0.50	0.6639	.....	1.349	0.0283	0.0447	1.58	0.07	Width of channel, 5 feet.

* The starred figures were used to obtain the depth on the weir given in column 5, and are not the results of experiment.

3d.—Weir without End Contraction. Heads taken at the Bottom of the Channel near the Weir.

Under this title it is proposed to show the difference between heads taken at the surface, 6 feet above the weir, and those taken at the bottom of the channel near the weir. This is but an indirect method of correcting for the effect of velocity of approach, but it shows clearly the relation existing between heads taken at the two places, and it allows the results to be presented with comparatively little labor.

In nearly every experiment the head taken near the weir by gauge No. 1, exceeded that taken by gauge No. 2.

The triangular space between the bottom of the channel and the face of the weir, where this excess of head exists, we have termed the angle of increased pressure, or, more briefly, the angle of pressure.

With each depth of channel, it was found that this excess of head near the weir was very nearly a constant proportion of the head due to the mean velocity of approach.

The formula adopted to represent this case is $E = ch$, in which,

E = the excess of the head taken at the bottom of the channel, near the weir, over the head taken near the surface, 6 feet from the weir.

c = a co-efficient which is constant for any given depth of channel.

h = the theoretical head due to the mean velocity of approach.

The co-efficients for the depths of channel experimented upon are as follows:

When	the	depth	of	channel	below	crest	of	weir	is	3.56	feet	$c =$	0.80
"	"	"	"	"	"	"	"	"	"	2.60	"	$c =$	0.62
"	"	"	"	"	"	"	"	"	"	1.70	"	$c =$	0.54
"	"	"	"	"	"	"	"	"	"	1.00	"	$c =$	0.50
"	"	"	"	"	"	"	"	"	"	0.50	"	$c =$	0.36

The formula, when applied to the smallest depth of channel, is only approximate.

Table IV. (pages 25 to 28) shows the result of each of the experiments.

TABLE IV.

Comparison of Heads on the Weir taken by Gauge No. 1 at the bottom of the channel near the Weir, and by Gauge No. 2 near the surface 6 feet above the Weir. Weir without end contractions, 5 feet long.

DEPTH OF CHANNEL OF APPROACH BELOW CREST OF WEIR, 3.56 FEET.

1 Number of the Experiment.	2 Date of the Experiment. — March, 1878.	3 Depth on Weir taken near the Surface 6 feet from the Weir. Gauge No. 2.	4 Theoretical Head due to Mean Velocity of Approach. h.	5 Excess of Head taken by Gauge No. 1 over Head taken by Gauge No. 2, as observed.	6 Excess of Head in angle of increased pressure near the Weir, as calculated by formula $E=0.8\,h$.	7 Difference of Observed from Calculated Excess.
		Feet.	Feet.	Feet.	Feet.	Feet.
1	21st.	0.1509	0.0001	0.0002	0.0001	+ 0.0001
2	"	0.1509	0.0001	0.0003	0.0001	+ 0.0002
3	16th.	0.1926	0.0001	0.0004	0.0001	+ 0.0003
4	"	0.1935	0.0001	—0.0001	0.0001	— 0.0002
5	21st.	0.2303	0.0002	0.0002	0.0002	0.0000
6	"	0.2304	0.0002	0.0002	0.0002	0.0000
7	16th.	0.2684	0.0002	0.0004	0.0002	+ 0.0002
8	"	0.2685	0.0002	0.0005	0.0002	+ 0.0003
9	12th.	0.3360	0.0004	0.0007	0.0003	+ 0.0004
10	"	0.3362	0.0004	0.0006	0.0003	+ 0.0003
11	21st.	0.3368	0.0004	0.0003	0.0003	0.0000
12	"	0.3369	0.0004	0.0005	0.0003	+ 0.0002
13	16th.	0.3566	0.0005	0.0005	0.0004	+ 0.0001
14	"	0.3574	0.0005	0.0006	0.0004	+ 0.0002
15	"	0.4216	0.0008	0.0004	0.0006	— 0.0002
16	"	0.4222	0.0008	0.0006	0.0006	0.0000
17	21st.	0.4244	0.0008	0.0009	0.0006	+ 0.0003
18	"	0.4245	0.0008	0.0009	0.0006	+ 0.0003
19	15th.	0.4260	0.0009	0.0006	0.0007	— 0.0001
20	"	0.4266	0.0009	0.0006	0.0007	— 0.0001
21	22d.	0.4302	0.0009	0.0011	0.0007	+ 0.0004
22	"	0.4308	0.0009	0.0010	0.0007	+ 0.0003
23	16th.	0.4926	0.0013	0.0011	0.0010	+ 0.0001
24	"	0.4939	0.0013	0.0007	0.0010	— 0.0003
25	22d.	0.5115	0.0014	0.0017	0.0011	+ 0.0006
26	"	0.5117	0.0014	0.0014	0.0011	+ 0.0003
27	12th.	0.5148	0.0015	0.0016	0.0012	+ 0.0004
28	"	0.5149	0.0015	0.0013	0.0012	+ 0.0001
29	22d.	0.5477	0.0017	0.0018	0.0014	+ 0.0004
30	20th.	0.5615	0.0018	0.0017	0.0014	+ 0.0003
31	"	0.5625	0.0019	0.0014	0.0015	— 0.0001
32	21st.	0.6002	0.0022	0.0015	0.0018	— 0.0003
33	"	0.6010	0.0022	0.0019	0.0018	+ 0.0001
34	"	0.6011	0.0022	0.0026	0.0018	+ 0.0008
35	19th.	0.6143	0.0024	0.0022	0.0019	+ 0.0003
36	12th.	0.6735	0.0030	0.0024	0.0024	0.0000
37	"	0.6748	0.0030	0.0028	0.0024	+ 0.0004
38	22d.	0.6897	0.0032	0.0025	0.0026	— 0.0001

TABLE IV.—(*Continued.*)

DEPTH OF CHANNEL OF APPROACH BELOW CREST OF WEIR, 3.56 FEET.

1	2	3	4	5	6	7
Number of the Experiment.	Date of the Experiment. —— March, 1878.	Depth on Weir taken near the Surface 6 feet from the Weir. Gauge No. 2.	Theoretical Head due to Mean Velocity of Approach. h.	Excess of Head taken by Gauge No. 1 over Head taken by Gauge No. 2, as observed.	Excess of Head in angle of increased pressure near the Weir, as calculated by formula $E=0.8\,h$.	Difference of Observed from Calculated Excess.
		Feet.	Feet.	Feet.	Feet.	Feet.
39	22d.	0.6924	0.0032	0.0026	0.0026	0.0000
40	"	0.6925	0.0032	0.0024	0.0026	— 0.0002
41	19th.	0.6962	0.0033	0.0021	0.0026	— 0.0005
42	"	0.6980	0.0033	0.0020	0.0026	— 0.0006
43	15th.	0.7153	0.0036	0.0023	0.0029	— 0.0006
44	18th.	0.7747	0.0044	0.0028	0.0035	— 0.0007
45	"	0.7759	0.0044	0.0027	0.0035	— 0.0008
46	13th.	0.7859	0.0046	0.0043	0.0037	+ 0.0006
47	22d.	0.8047	0.0049	0.0041	0.0039	+ 0.0002
48	20th.	0.8310	0.0053	0.0039	0.0042	— 0.0003
49	"	0.8336	0.0053	0.0041	0.0042	— 0.0001
50	11th.	0.8651	0.0059	0.0051	0.0047	+ 0.0004
51	20th.	0.8806	0.0062	0.0040	0.0050	— 0.0010
52	"	0.8812	0.0062	0.0045	0.0050	— 0.0005
53	14th.	0.9238	0.0070	0.0043	0.0056	— 0.0013
54	22d.	0.9437	0.0075	0.0047	0.0060	— 0.0013
55	"	0.9449	0.0075	0.0047	0.0060	— 0.0013

DEPTH OF CHANNEL OF APPROACH BELOW CREST OF WEIR, 2.60 FEET.

$E=0.62\,h$.

56	16th.	0.2685	0.0004	0.0004	0.0002	+ 0.0002
57	12th.	0.3359	0.0008	0.0005	0.0005	0.0000
58	16th.	0.3564	0.0009	0.0007	0.0006	+ 0.0001
59	"	0.4206	0.0015	0.0009	0.0009	0.0000
60	15th.	0.4253	0.0015	0.0009	0.0009	0.0000
61	16th.	0.4925	0.0022	0.0015	0.0014	+ 0.0001
62	12th.	0.5132	0.0025	0.0021	0.0016	+ 0.0005
63	20th.	0.5598	0.0032	0.0021	0.0020	+ 0.0001
64	19th.	0.6119	0.0040	0.0031	0.0025	+ 0.0006
65	12th.	0.6703	0.0051	0.0046	0.0032	+ 0.0014
66	19th.	0.6919	0.0056	0.0033	0.0035	— 0.0002
67	15th.	0.7117	0.0059	0.0034	0.0037	— 0.0003
68	18th.	0.7716	0.0073	0.0043	0.0045	— 0.0002
69	20th.	0.8284	0.0088	0.0049	0.0055	— 0.0006
70	11th.	0.8595	0.0097	0.0067	0.0060	+ 0.0007
71	20th.	0.8758	0.0101	0.0059	0.0063	— 0.0004
72	14th.	0.9180	0.0114	0.0069	0.0071	— 0.0002

TABLE IV.—(*Continued.*)

DEPTH OF CHANNEL OF APPROACH BELOW CREST OF WEIR, 1.70 FEET.

1	2	3	4	5	6	7
Number of the Experiment.	Date of the Experiment. —— March, 1878.	Depth on Weir taken near the Surface 6 feet from the Weir. Gauge No. 2.	Theoretical Head due to Mean Velocity of Approach. h.	Excess of Head taken by Gauge No. 1 over Head taken by Gauge No. 2, as observed.	Excess of Head in angle of increased pressure near the Weir, as calculated by formula $E = 0.54\ h$.	Difference of Observed from Calculated Excess.
		Feet.	Feet.	Feet.	Feet.	Feet.
73	16th.	0.1926	0.0004	0.0004	0.0002	+ 0.0002
74	"	0.2676	0.0009	0.0005	0.0005	0.0000
75	12th.	0.3341	0.0016	0.0012	0.0009	+ 0.0003
76	16th.	0.3551	0.0019	0.0008	0.0010	— 0.0002
77	"	0.4183	0.0030	0.0012	0.0016	— 0.0004
78	15th.	0.4232	0.0030	0.0015	0.0016	— 0.0001
79	16th.	0.4887	0.0044	0.0021	0.0024	— 0.0003
80	12th.	0.5088	0.0050	0.0031	0.0027	+ 0.0004
81	20th.	0.5551	0.0062	0.0031	0.0033	— 0.0002
82	19th.	0.6057	0.0077	0.0043	0.0042	+ 0.0001
83	12th.	0.6627	0.0097	0.0055	0.0052	+ 0.0003
84	19th.	0.6836	0.0106	0.0051	0.0057	— 0.0006
85	15th.	0.7018	0.0113	0.0060	0.0061	— 0.0001
86	18th.	0.7606	0.0137	0.0070	0.0074	— 0.0004
87	13th.	0.7695	0.0142	0.0085	0.0077	+ 0.0008
88	20th.	0.8150	0.0163	0.0083	0.0088	— 0.0005
89	11th.	0.8467	0.0178	0.0103	0.0096	+ 0.0007
90	20th.	0.8595	0.0187	0.0098	0.0101	— 0.0003
91	14th.	0.9009	0.0209	0.0116	0.0113	+ 0.0003
92	22d.	0.9215	0.0220	0.0114	0.0119	— 0.0005
93	"	0.9223	0.0221	0.0124	0.0119	+ 0.0005

DEPTH OF CHANNEL OF APPROACH BELOW CREST OF WEIR, 1.00 FEET.

$E = 0.50\ h$.

94	16th.	0.1914	0.0009	0.0005	0.0005	0.0000
95	"	0.2649	0.0022	0.0012	0.0011	+ 0.0001
96	12th.	0.3308	0.0038	0.0021	0.0019	+ 0.0002
97	16th.	0.3496	0.0044	0.0022	0.0022	0.0000
98	"	0.4105	0.0067	0.0032	0.0034	— 0.0002
99	15th.	0.4155	0.0068	0.0035	0.0034	+ 0.0001
100	16th.	0.4784	0.0097	0.0046	0.0049	— 0.0003
101	12th.	0.4965	0.0108	0.0051	0.0054	— 0.0003
102	20th.	0.5409	0.0132	0.0065	0.0066	— 0.0001
103	19th.	0.5901	0.0162	0.0084	0.0081	+ 0.0003
104	12th.	0.6443	0.0201	0.0108	0.0101	+ 0.0007
105	19th.	0.6655	0.0217	0.0103	0.0109	— 0.0006
106	15th.	0.6806	0.0230	0.0120	0.0115	+ 0.0005
107	18th.	0.7378	0.0275	0.0145	0.0138	+ 0.0007
108	20th.	0.7873	0.0322	0.0160	0.0161	— 0.0001
109	"	0.8293	0.0366	0.0179	0.0183	— 0.0004
110	14th.	0.8665	0.0406	0.0222	0.0203	+ 0.0019
111	22d.	0.8854	0.0426	0.0205	0.0213	— 0.0008

TABLE IV.—(*Continued.*)

DEPTH OF CHANNEL OF APPROACH BELOW CREST OF WEIR, 0.50 FEET.

1 Number of the Experiment.	2 Date of the Experiment. — March, 1878.	3 Depth on Weir taken near the Surface 6 feet from the Weir. Gauge No. 2.	4 Theoretical Head due to Mean Velocity of Approach. h.	5 Excess of Head taken by Gauge No. 1 over Head taken by Gauge No. 2, as observed.	6 Excess of Head in angle of increased pressure near the Weir, as scaled from curve sho'g mean of experiments.	7 Difference of observed excess from excess obtained by diagram.	8 Excess of Head in dead angle by approximate formula $E=0.36\,h$.
		Feet.	Feet.	Feet.	Feet.	Feet.	Feet.
112	16th.	0.1884	0.0027	0.0011	0.0011	0.0000	0.0010
113	"	0.2594	0.0060	0.0026	0.0026	0.0000	0.0022
114	"	0.3404	0.0114	0.0046	0.0048	— 0.0002	0.0041
115	"	0.3974	0.0165	0.0064	0.0066	— 0.0002	0.0059
116	"	0.4595	0.0230	0.0090	0.0088	+ 0.0002	0.0083
117	20th.	0.5186	0.0302	0.0110	0.0112	— 0.0002	0.0109
118	21st.	0.5509	0.0345	0.0126	0.0127	— 0.0001	0.0124
119	18th.	0.6926	0.0583	0.0224	0.0201	+ 0.0023	0.0210
120	20th.	0.7402	0.0667	0.0233	0.0231	+ 0.0002	0.0240
121	"	0.7764	0.0752	0.0238	0.0256	— 0.0018	0.0271
122	22d.	0.8269	0.0860	0.0296	0.0296	0.0000	0.0310
*123	12th.	0.3188	0.0100	0.0045	0.0041	+ 0.0004	0.0036
*124	15th.	0.3996	0.0169	0.0067	0.0066	+ 0.0001	0.0061
*125	12th.	0.4707	0.0256	0.0109	0.0092	+ 0.0017	0.0092
*126	19th.	0.5681	0.0360	0.0083	0.0135	— 0.0052	0.0130
*127	12th.	0.6182	0.0435	— 0.0036	0.0160	— 0.0196	0.0157
*128	19th.	0.6358	0.0466	0.0077	0.0169	— 0.0092	0.0168
*129	15th.	0.6412	0.0499	0.0172	0.0172	0.0000	0.0180

Columns 5 and 6 of the table show the excess of head taken at the bottom of the channel near the weir, over the head taken near the surface 6 feet from the weir; the first shows the results of observations, the second the results of calculations by using the formula $E = ch$, with the co-efficients given on page 24.† The differences between these results are given in feet in column 7, and an examination of this column shows the following features:

In the first 55 experiments which were made with the bottom of the channel 3.56 feet below the crest of the weir, the differences are small, yet it may be observed that in the first portion of the experiments they are above, while in the latter portion they are below the observed results. This indicates inaccuracy in the formula, or some slight cause affecting many of the observations, but the maximum difference within the limits

* Defective experiments, corresponding to those marked with an asterisk in Table II.

† The quantities in column 6 on this page are obtained from a diagram.

of the experiments being only 0.0013 feet, it was not thought necessary to seek a formula which would agree more closely. When the depth of channel was 2.60 feet, as in Exp. 56 to 72, the formula agreed with the experiments rather more closely. In Experiments 73 to 93, and 94 to 111, in which the depths of the channel were respectively 1.70 and 1.00 feet, the differences between the calculated and observed results are small, and change their sign frequently, thus showing a close accordance between the formula and experiments, which is the more marked on account of the increased size of the quantities given in columns 5 and 6. It is somewhat remarkable in this case, that so simple a formula should give so accurate results.

When the depth of the channel was but 0.50 feet, no accurate formula was determined; the experiments Nos. 112 to 122 were plotted, and a regular curve was drawn to represent them as nearly as possible. This curve has been used in the place of a formula, to obtain the values given in column 6. Column 8 has been added to show the excess of head near the weir as calculated by the approximate formula given on page 24. A comparison of the two columns (6 and 8) does not exhibit any large differences, showing that the excess of head near the weir follows nearly the same laws as when the depth of channel is much greater.

Experiments 123 to 129 were defective, and correspond to the experiments marked with an an asterisk in Table II. As the differences given in column 7 are generally small, a description of some of the precautions taken to obtain these results may not be uninteresting. Experiments of this kind depend chiefly upon the measurement of the relative heights of the water in the hook-gauge pails. The comparative heights of the gauges were measured nearly every day, by taking gauge readings when the weir basin was full, but when little or no water was running over the weir. The instability of the two hook-gauges with reference to each other was largely prevented by fastening them to the same post. Personal error (which some experiments showed to be a cause of inaccuracy in gauge readings) was eliminated by having each assistant read the same gauge during the comparison of the level of the gauges, as well as during the experiments. The graduation of the gauges was compared, and found to vary not more than 0.0001 feet.

4TH.—WEIR WITH END CONTRACTION. HEADS TAKEN AT THE BOTTOM OF THE CHANNEL NEAR THE WEIR.

In this case, the head taken near the weir was generally greater than when taken 6 feet up stream, though in a few experiments, under extreme conditions, it was less. The formula $E = ch$, which was found so applicable in the case last considered, applies in this case only when the depth of channel is large in comparison with the depth on the weir.

For the smaller depths of channel, the co-efficient varies much with the depth on the weir, with the proportion of the width of the channel occupied by weir, and with other causes.

When the depth of the channel below the crest of the weir is 3.56 feet, the co-efficient, within the limits of the experiments, is unity, and the formula becomes $E = h$.

Table V. (pages 31–32) shows the result of each of the experiments made with a depth of channel of 3.56 feet. The table is divided into four parts, corresponding to the different forms of weirs experimented upon ; its contents are sufficiently explained by the headings.

An examination of column 6 shows that with all, except the larger depths on the weir, the difference of the observed excess of head from the theoretical head due to the mean velocity of approach is generally positive, indicating some slight cause of error, or a co-efficient which exceeds unity. With the larger depths the differences are in the other direction, and are comparatively large, indicating that in these cases the co-efficient should be reduced.

Within the limits of the experiments, the differences are not very large, and the formula as given may be used, but it would not be safe to extend it to greater depths on the weir, though it is quite probable that it would apply well to greater depths of channel.

TABLE V.

Comparison of Heads on the Weir, taken at Gauge No. 1, at the bottom of the channel near the Weir, and Gauge No. 2, near the surface 6 feet above the Weir. Weir with end contraction. Bottom of channel 3.56 feet below Crest of Weir. Width of channel, 5 feet.

ONE END CONTRACTION—WEIR 4 FEET LONG.

1	2	3	4	5	6
Number of the Experiment.	Date of the Experiment. — March, 1878.	Depth on the Weir, taken near the Surface 6 feet from the Weir. Gauge No. 2.	Excess of Head in Angle of Increased Pressure near Weir, as observed.	Theoretical Head due to Mean Velocity of Approach. h.	Difference of Observed Excess from Theoretical Head given in Column 5.
		Feet.	Feet.	Feet.	Feet.
1	21st.	0.1761	0.0002	0.0000	+0.0002
2	"	0.1763	0.0002	0.0000	+0.0002
3	"	0.2691	0.0002	0.0001	+0.0001
4	"	0.2693	0.0003	0.0001	+0.0002
5	"	0.3942	0.0003	0.0004	−0.0001
6	"	0.3944	0.0003	0.0004	−0.0001
7	"	0.4978	0.0008	0.0008	0.0000
8	22d.	0.5996	0.0017	0.0014	+0.0003
9	"	0.5997	0.0015	0.0014	+0.0001
10	21st.	0.7055	0.0023	0.0021	+0.0002
11	"	0.7064	0.0030	0.0021	+0.0009
12	22d.	0.9432	0.0036	0.0046	−0.0010
13	"	0.9448	0.0033	0.0046	−0.0013
14	"	0.9460	0.0029	0.0046	−0.0017

ONE END CONTRACTION—WEIR 3.31 FEET LONG.

15	21st.	0.4498	0.0004	0.0004	0.0000
16	"	0.5678	0.0008	0.0008	0.0000
17	22d.	0.5764	0.0009	0.0008	+0.0001
18	"	0.5766	0.0012	0.0008	+0.0004
19	"	0.6860	0.0015	0.0013	+0.0002
20	21st.	0.8062	0.0024	0.0020	+0.0004
21	"	0.8063	0.0017	0.0020	−0.0003
22	22d.	0.9297	0.0022	0.0029	−0.0007
23	"	0.9317	0.0026	0.0029	−0.0003

TABLE V.—(*Continued.*)

TWO END CONTRACTIONS—WEIR 3 FEET LONG.

1 Number of the Experiment.	2 Date of the Experiment. —— March, 1878.	3 Depth on the Weir, taken near the Surface 6 feet from the Weir. Gauge No. 2	4 Excess of Head in Angle of Increased Pressure near Weir, as observed.	5 Theoretical Head due to Mean Velocity of Approach. h.	6 Difference of Observed Excess from Theoretical Head given in Column 5.
		Feet.	Feet.	Feet.	Feet.
24	21st.	0.2155	0.0001	0.0000	+0 0001
25	"	0.3301	0.0002	0.0001	+0.0001
26	"	0.4843	0.0003	0.0004	—0.0001
27	22d.	0.6215	0.0010	0.0008	+0.0002
28	"	0.7398	0.0015	0.0013	+0.0002
29	21st.	0.8702	0.0014	0.0020	—0.0006
30	"	0.8714	0.0024	0.0020	+0.0004

TWO END CONTRACTIONS—WEIR 2.31 FEET LONG.

31	21st.	0.5824	0.0003	0.0004	—0.0001
32	22d.	0.7478	0.0011	0.0008	+0.0003
33	"	0.8905	0.0014	0.0012	+0.0002
34	"	0.9548	0.0013	0.0014	—0.0001

Table VI. (page 33) shows the result of all of the experiments, where the depth of the channel below the crest of the weir was 2.60 feet, or less. Column 6 of this table shows the co-efficient, c, in the formula, $E=ch$, as deduced from each experiment.

An examination of this column shows that the co-efficients are very variable, ranging from + 0.87 in Exp. 1, to — 0.13 in Exp. 12.

A comparison of Experiments 4, 5 and 7; or Experiments 8, 10 and 12, shows that, other things being equal, an increase in depth on the weir causes a decrease in the co-efficient.

The experiments given in this table do not furnish a sufficient basis for the determination of a formula to represent them; and their chief value is in the warning they give, that, under such circumstances, the head should not be taken in the angle near the weir.

TABLE VI.

Comparison of Heads on the Weir taken at Gauge No. 1, at the bottom of the channel near the Weir, and at Gauge No. 2, near the surface 6 feet above the Weir. Weir with end contraction. Width of channel, 5 feet.

BOTTOM OF CHANNEL OF APPROACH BELOW CREST OF WEIR 2.60 FEET.

1	2	3	4	5	6	7	8
No. of the Experiment.	Date of the Experiment. — March. 1878.	Depth on the Weir, taken near the surface 6 feet from the Weir. Gauge No. 2.	Theoretical head due to mean velocity of approach. h.	Excess of head in angle of increased pressure near Weir, as observed.	Value of c in the formula $E = ch$.	Number of End Contractions.	Length of Weir.
1	21st.	0.8036	0.0033	0.0029	0.87	1	3.31

BOTTOM OF CHANNEL 1.70 FEET BELOW WEIR.

2	21st.	0.7969	0.0062	0.0033	0.54	1	3.31
3	"	0.8612	0.0058	0.0031	0.53	2	3.00

BOTTOM OF CHANNEL 1.00 FOOT BELOW WEIR.

4	22d.	0.5647	0.0058	0.0017	0.30	1	3.31
5	21st.	0.7850	0.0120	0.0031	0.26	1	3.31
6	"	0.8490	0.0112	0.0042	0.37	2	3.00
7	22d.	0.9024	0.0162	0.0037	0.23	1	3.31

BOTTOM OF CHANNEL 0.50 FOOT BELOW WEIR.

8	22d.	0.5574	0.0126	0.0001	0.01	1	3.31
9	21st.	0.6639	0.0283	0.0064	0.23	1	4.00
10	"	0.7677	0.0239	— 0.0022	—0.09	1	3.31
11	"	0.8310	0.0217	— 0.0009	—0.04	2	3.00
12	22d.	0.8775	0.0309	— 0.0041	—0.13	1	3.31

Miscellaneous Experiments relating to "Velocity of Approach."

In all the experiments before described the screens were quite distant from the weir, and many precautions were taken to render the velocity in all parts of the channel as uniform as possible; but many points came up during the observations, or when studying their results, which led us to consider whether or to what extent the conclusions arrived at would be applicable to other cases in which the conditions of the flow of water are not the same as for the apparatus experimented upon.

Therefore some additional investigations were made to elucidate the following questions:

1st. Whether the head taken through the orifice at the side of the weir (Gauge No. 2) represents truly the height of the surface of the water over the orifice?

2d. How uniform were the velocities in different parts of the channel of approach during the experiments?

3d. Whether the effect of velocity of approach would be modified if the screens were placed nearer the weir?

4th. Whether the effect of velocity of approach would be modified by irregular velocities in the channel?

5th. Having found by the experiments already considered that the head taken at the bottom of the channel near the weir, by Gauge No. 1, is greater than that taken 6 feet from the weir, by Gauge No. 2, how far does the angle, in which this excess of head exists, extend; and what would be the result if the head was measured at other points than those used? also, how far up-stream does the curvature of the sheet, passing the weir, extend?

6th. What are the motions of the water on the up-stream side of the weir?

7th. How is the curvature of the sheet passing the weir modified by the velocity of approach?

1st. Mr. Mills, from his numerous and accurate experiments upon piezometers,* concludes that, with currents flowing parallel with the side of a straight conduit where the velocity is neither increasing nor diminishing, with orifices having edges in the plane of the side

* Experiments upon Piezometers used in Hydraulic Investigations, by Hiram F. Mills, C. E., p. 52, Proceedings of the American Academy of Arts and Sciences, 1878.

and with passages normal thereto, the piezometric column will indicate the true height of the surface of the water in the conduit when in motion as well as when at rest. He further shows the necessity of attending carefully to the conditions given above, when it is desired to obtain exact results. In arranging the apparatus for the experiments on velocity of approach these conditions were carefully attended to; but that there might be no doubt about the result obtained, the height of the surface of the water was measured and compared with the head taken through the orifice. To measure the height of the water a gauge was provided similar in construction to a hook-gauge, except that the lower end of the movable rod terminated in a sharp point, instead of being bent into a hook. When taking an observation the point was lowered until it touched the surface of the water. The measurement was taken about 0.4 feet from the side of the channel, and directly opposite the orifice. The levels of the point and hook gauges were compared by taking measurements when the water in the weir basin was at rest.

Table VII. gives the results of the experiments.

TABLE VII.

Comparison of the Head taken at the orifice in the side of the channel by Gauge No. 2, with the height of the surface of the water above the orifice, as measured with the point gauge.

	Experiment 1, March 18, 1878.				Experiment 2, March 19, 1878.			
Depth of Channel below Crest of Weir.	Theoretical Head due to Mean Velocity of Approach.	Head on Weir, taken at Orifice in Side of Channel.	Depth on Weir, as measured from the Surface of Water 6 feet from the Weir.	Difference of Quantity in Column 3 from Quantity in Column 4.	Theoretical Head due to Mean Velocity of Approach.	Head on Weir, taken at Orifice in Side of Channel.	Depth on Weir, as measured from the Surface of Water 6 feet from the Weir.	Difference of Quantity in Column 7 from Quantity in Column 8.
1	2	3	4	5	6	7	8	9
Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	
3.56	0.0044	0.7750 0.7762	0.7750 0.7756	—0.0003	0.0033	0.6975 0.6957	0.6974 0.6958	—0.0
2.60	0.0073	0.7716	0.7712	—0.0004	0.0056	0.6919	0.6918	—0.0001
1.70	0.0137	0.7606	0.7606	0.0000	0.0106	0.6836	0.6844	+0.0008
1.00	0.0275	0.7378	0.7381	+0.0003	0.0217	0.6655	0.6659	+0.0004
0.50	0.0583	0.6926	0.6930	+0.0004	0.0466	*0.6270	0.6283	+0.0013

* This height was obtained from the head taken by Gauge No. 1, as the head taken at the orifice was incorrect; this being one of the defective experiments referred to on page 13.

The differences shown in columns 5 and 9 may be due either to inaccuracy in the measurement, or to the effect of the velocity of the water. If to the latter cause, they should be proportional to the head due to the velocity. A comparison of the heads in columns 2 and 6, with the differences in columns 5 and 9, does not show any marked relation between them.

The slight oscillations of the surface of the water in the channel did not allow the point-gauge measurements to be taken with the same degree of accuracy as those taken with the hook-gauge.

An examination of the experiments led to the conclusion that the differences are no greater than may be attributed to error in measurement; but even if they were caused by the velocity of the water passing the orifice, the effect would not then exceed 0.02 h, and, consequently, would not modify, to a greater extent, the co-efficients of the formulæ which have been given.

2d. The velocities in different parts of the channel of approach were measured with a current meter.

The first experiment was made before the screens were placed across the channel, and consisted of nine measurements in the cross-section of the channel, 23 feet from the weir, and a similar number 10 feet from the weir. The velocities are given in Table VIII., and are shown in their relative positions. They were taken 6 inches below the surface, 6 inches above the bottom, at mid-depth, 9 inches from either side, and in the middle of the channel.

TABLE VIII.

VELOCITIES IN CHANNEL OF APPROACH.—MARCH 9, 1878.

23 FEET FROM WEIR.			10 FEET FROM WEIR.		
0.377	0.662	0.464	0.548	0.562	0.485
0.690	0.683	0.598	0.605	0.591	0.598
0.712	0.690	0.626	0.562	0.555	0.612

Depth of channel below crest of weir, 3.56 feet.

Depth on weir, 0.786 feet.

Mean velocity of approach by weir measurement, 0.54 feet per second.

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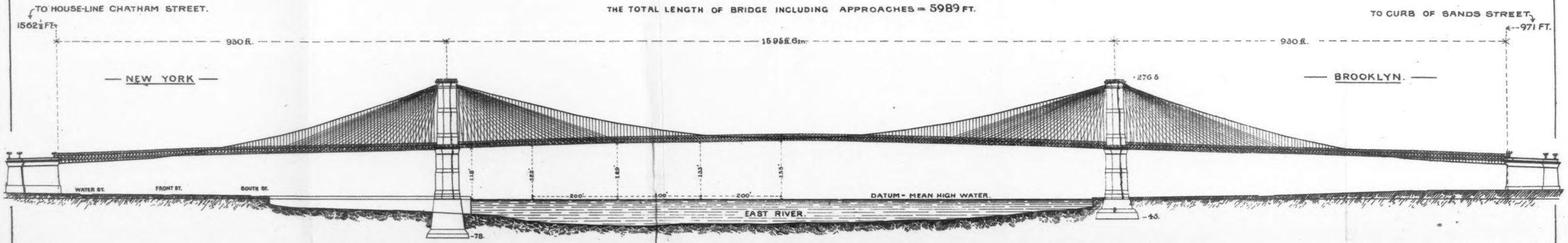
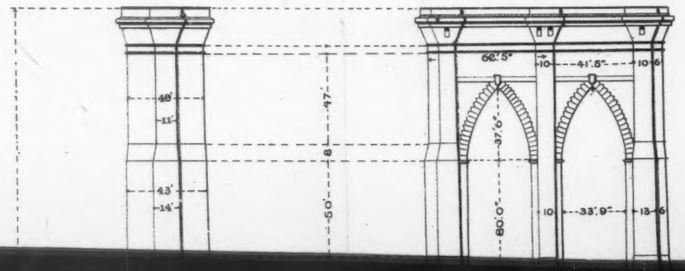
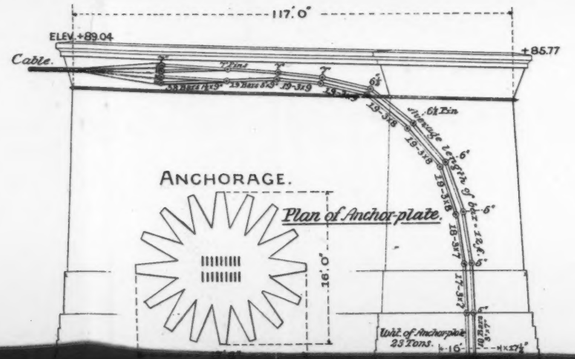
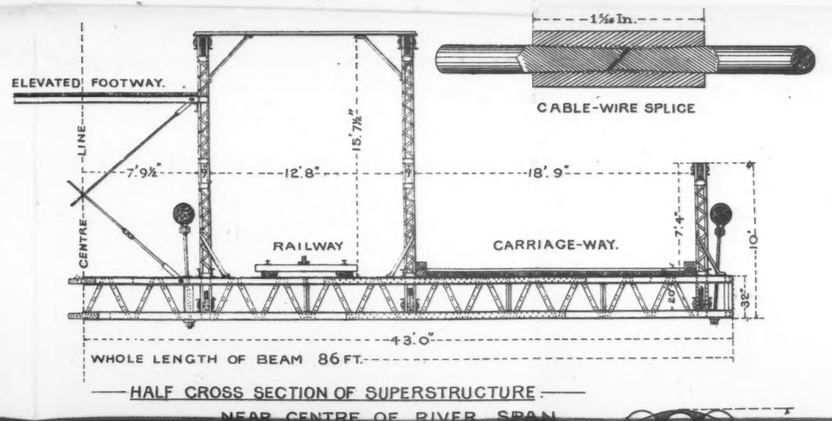


TOWERS.





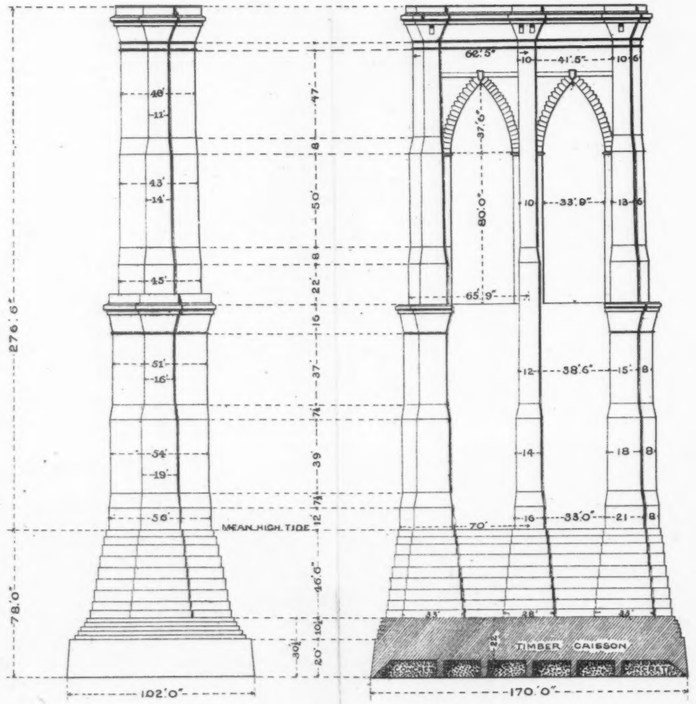


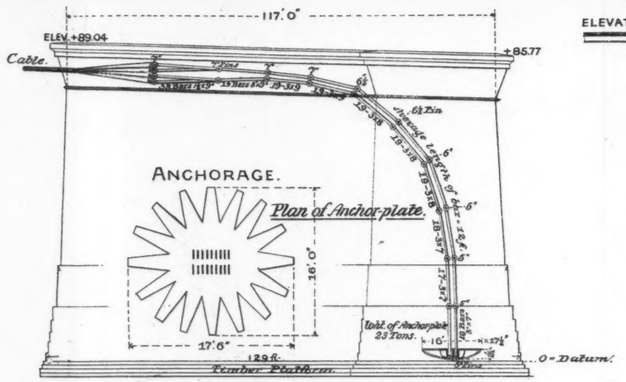
HALF CROSS SECTION OF SUPERSTRUCTURE
NEAR CENTRE OF RIVER SPAN.

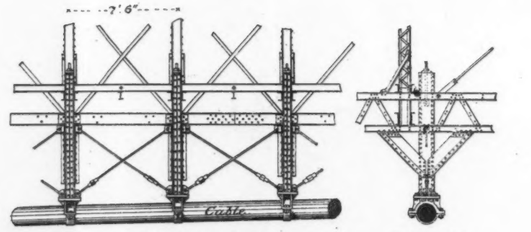
TOWERS.

END VIEW.

FRONT ELEVATION.



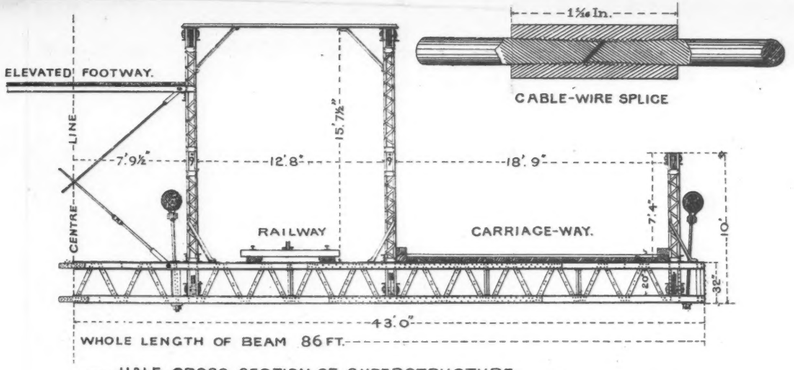




SHOWING FLOOR ABOVE CABLE NEAR ANCHORAGE.

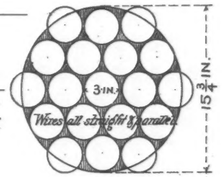
Scale of Truss Details

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 Ft.



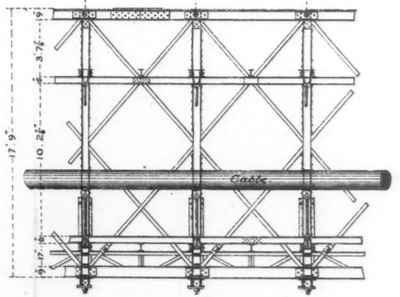
HALF CROSS SECTION OF SUPERSTRUCTURE NEAR CENTRE OF RIVER SPAN.

CABLE-WIRE SPLICE

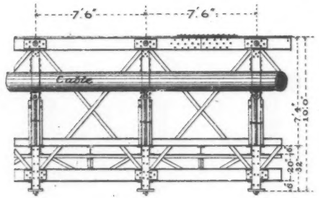


SECTION OF CABLE AND ARRANGEMENT OF STRANDS.

Total Wires in One Cable = 5296.



ELEVATION INNER TRUSS



OUTER TRUSS

The velocities 23 feet from the weir are quite irregular, which may be due in part to contraction at the entrance of the channel. The velocities 10 feet from the weir are much more uniform.

Other measurements were taken at different depths in the middle of the channel, and at short intervals along its axis.

A portion of them are given in Table IX.

TABLE IX.

VELOCITIES IN CHANNEL OF APPROACH.—MARCH 13TH, 1878.

1 Position of Current Meter.	2 Velocity 21 ft. from Weir=3.3 ft. above the Screens. Ft. per sec.	3 Velocity 16 ft. from Weir=1.7 ft. below the Screens. Ft. per sec.	4 Velocity 8 ft. from the Weir. Ft. per sec.	5 REMARKS.
EXPERIMENT NO. 1.—FLOW EVENLY SCREENED.				
0.50 ft. below surface...	0.662	0.618	0.568	Depth of channel below crest of weir, 3.56 ft.
Mid-depth.....	0.598	0.654	0.647	Depth on weir, 0.79 ft.
0.46 ft. above bottom...	0.584	0.661	0.626	Mean velocity of approach, by weir measurement, 0.55 ft. per second.
EXPERIMENT NO. 2.—SCREENS RAISED 0.44 FEET ABOVE BOTTOM.				
0.50 ft. below surface...	0.584	0.549	0.560	Depth of channel below crest of weir, 3.56 ft.
Mid-depth.....	0.612	0.578	0.566	Depth on weir, 0.78 ft.
0.46 ft. above bottom...	0.591	0.633	0.629	Mean velocity of approach, by weir measurement, 0.54 ft. per second.
EXPERIMENT NO. 3.—FLOW EVENLY SCREENED.				
0.47 ft. below surface..	.....	1.080	1.089	Depth of channel below crest of weir, 1.70 ft.
Mid-depth.....	.....	1.100	1.140	Depth on weir, 0.78 ft.
0.40 ft. above bottom...	.....	0.954	1.052	Mean velocity of approach, by weir measurement, 0.96 ft. per second.

NOTE.—No measurements were taken above the screens in Experiment 3, because the bottom of the channel was inclined, as shown by the dotted lines at I (Plate II., Fig. 1).

The results in column 4 show that the large effect of the velocity of approach, as found in the main experiments, is not to be attributed to an excessive velocity in the channel, directly opposite the opening of the weir.

An examination of the experiments showed some other features of interest.

It is known that, at a sufficient distance from the entrance of a long uniform channel, the velocities, in different parts of any cross-section, are arranged in accordance with certain laws, the bottom velocities being the smallest. A comparison of columns 3 and 4 show, that eight feet (the distance between points of measurement) is not a sufficient distance to cause even a partial rearrangement, such as might be looked for.

A comparison of columns 2 and 3, in Experiment 1, shows that above the screens the velocity near the surface was greatest, while below the screens the greatest velocity was near the bottom, that is, the effect of screens in a uniform channel is to increase the bottom velocities.

The same comparison may be made in Experiment 2, but it has less value, as the flow was not evenly screened.

3d. Effect of placing the screens nearer the weir. The results are given in the following table:

TABLE X.

1	2	3	4	5	6
Distance from the Weir to the Screens.	Depth on Weir, as taken 6 feet from the Weir.	Depth on Weir, as taken near the Weir.	Difference in depth on the Weir due to changing the Screens from their first position.		Remarks
Feet.	Gauge 2. Feet.	Gauge 1. Feet.	Gauge 2. Feet.	Gauge 1. Feet.	
17.7	0.6924	0.6949	.....	.....	Depth of channel below crest of weir, 3.56 ft. Mean velocity of approach, 0.457 ft.
11.3	0.6927	0.6954	+0.0003	+0.0005	Theoretical head due to mean velocity of approach, 0.0033 ft.
7.3	0.6921	0.6945	—0.0003	—0.0004	Effect of velocity of approach upon depth on weir at Gauge 2, 0.0048 ft.

It may be seen from the table, that the maximum difference in depth over the weir, due to changing the screens from their first position, was 0.0005 feet, or about 10 per cent. of the total effect of velocity of

approach. The difference might equally well be attributed to a slight variation in the quantity of water passing the weir.

4th. To ascertain the effect of irregular velocities in the channel of approach, such irregularities were produced artificially by partially or wholly obstructing portions of the channel.

The results of the experiments are given in the following table:

TABLE XI.

1	2	3	4	5	6
No. of the Exp.	Depth on the Weir, as taken 6 feet above the Weir.	Depth on the weir, as taken near the Weir.	Difference between depths on the Weir due to irregular velocities.		Remarks.
	Gauge 2. Feet.	Gauge 1. Feet.	Gauge 2. Feet.	Gauge 1. Feet.	
			Series No. 1.		
	Depth of channel below crest of weir..... 3.56 feet.				
	Mean velocity of approach..... 0.617 "				
	Theoretical head due to mean velocity..... 0.0059 "				
	Effect of velocity of approach at Gauge 2..... 0.0085 "				
1	0.8651	0.8702	.....	.....	Flow evenly screened.
2	0.8652	0.8704	+0.0001	+0.0002	Screens removed.
3	0.8647	0.8723	—0.0004	+0.0021	Screens removed and the entrance of the channel obstructed by a board extending 1.90 ft. upward from the bottom.
4	0.8652	0.8743	+0.0001	+0.0041	Same as last case, except that the board extended 2 feet downward from the surface.
			Series No. 2.		
	Depth of channel below crest of weir..... 1.7 feet.				
	Mean velocity of approach..... 1.191 "				
	Theoretical head due to mean velocity..... 0.0221 "				
	Effect of velocity of approach at Gauge 2..... 0.0344 "				
5	0.9223	0.9347	.....	.....	Flow evenly screened.
6	0.9280	0.9422	+0.0057	+0.0075	7.3 ft. from the weir, the flow was partially obstructed by a screen extending 1.35 feet downward from the surface.
7	0.9147	0.9257	—0.0076	—0.0090	Same as last case, except that the screen extended 1.39 ft. upward from the bottom.

Upon examining the differences due to irregular velocities, it is very noticeable that they affect the measurement of the depth on the weir, as

taken by Gauge No. 1, to a much larger extent than when taken by Gauge No. 2.

The maximum difference in depth at Gauge No. 2, as given in column 4, is 0.0076 feet, equal to 0.34 h, or about 22 per cent. of the total effect of velocity of approach.

In column 5 (Gauge No. 1), the largest proportional difference is in Exp. 4, 0.0041 feet, and equals 0.70 h, or about 48 per cent. of the total effect of velocity of approach.

In these experiments the water approaching the weir was very turbulent, more so than often happens in practice. From these experiments it is thought that the observer may judge within what limits of accuracy the formulæ already given for velocity of approach are applicable to cases where the flow is irregular.

One other feature of the experiments deserves notice. In Exp. 4 the excess of head near the weir (the difference between columns 2 and 3) is 0.0091 feet, while the theoretical head due to the mean velocity of approach is but 0.0059 feet. The formula for this case, when the flow is regular, is $E = 0.80\ h$ (see p. 24). To represent Exp. 4, the formula becomes $E = 1.54\ h$, *i. e.*, the co-efficient is greater than unity.

5th. Comparison of the heads taken at various points above the weir, with the head at the orifice in the side of the channel, 6 feet above the weir.

To make this comparison, one end of the flexible pipe, leading to Gauge No. 2, was passed through the side partition into the channel of approach, and connected with a short piece of gas pipe, to which handles were attached, so that its open end could be held in any desired position.

Mr. Mills' experiments* show, that when a pipe having a square end is held at right angles to a current, the pressure of the water in the pipe is diminished by a portion of the head due to the velocity of the water passing the end of the pipe.

He attributes this to the fact that the particles of water, which would pass where the pipe is, are deviated from their course, and a part of them moving lengthwise of the pipe, are projected in a curve around its end, thus causing the diminished pressure. The action just described was prevented in the following manner:

A steel plate 0.66 feet long, 0.50 feet wide, and 0.003 feet thick, with

* Experiments on Piezometers, page 51.

an orifice in the centre 0.037 feet in diameter, was fastened to the end of the gas pipe, and at right angles with it. The edges of the plate were beveled on the back side. The face of the plate was straight and smooth, and the edges of the orifice sharp, square and flush with the plate.

When taking measurements the plate was held in a vertical plane, and in the line of the current. Simultaneous measurements were taken with both gauges, one showing the head at the orifice in the movable plate, and the other the head at the pipe near the weir.*

The experiments were discontinued while yet incomplete, on account of the time required to adjust the plate in position, and because the plate could not be set accurately in the line of the current when at the lower depths. The experiments are shown on Plate IV. The position of the pipe near the weir is shown at *B*; the position of the orifice in the side of the channel at *A*. The crosses indicate the various positions of the orifice in the steel plate.

The figures show the heads taken at the various points, as compared with the head at *A*. The broken line shows approximately the limit of the angle where the head is greater than at *A*.

It may be noticed that the greatest excess of head is near the weir, about 1.5 feet above the bottom of the channel.

A series of experiments was made to determine the extent of the angle of pressure by measuring with a current meter the velocities at different distances above the weir, and at different levels. The application of these experiments to the subject under consideration is found in the fact that increase of velocity being the result of loss of head, and decrease in velocity a cause of increased head or pressure, the point above the weir where the bottom velocities begin to decrease is the limit of the angle where the excess of head exists.

At the surface, the point where the velocities begin to increase, is also the up-stream limit of the curve of the sheet flowing over the weir. All of the measurements were made in the middle of the channel.

Two experiments were made, in which the depth of the channel below the crest was 3.56 feet, and the depth on the weir was about 0.79 feet.

The centre of the current meter, which was provided with a wheel

* The fixtures of Gauge No. 2 being used in connection with the movable plate, the comparison was made with Gauge No. 1; and the head at the orifice, 6 ft. above the weir, was obtained by calculation from the former.

0.4 feet in diameter, was held 0.46 feet above the bottom of the channel, 0.50 feet below the surface, and at mid-depth.

Commencing at the head of the channel, the velocities near the bottom began to diminish gradually about eight feet from the weir; five feet from the weir they had decreased about ten per cent., and two feet from the weir about thirty per cent.

The velocities near the surface began to increase very gradually about seven feet from the weir; at five feet they had increased about five per cent.; at two feet from the weir they had increased about seventy-five per cent., and were increasing very rapidly.

The mid-depth velocities began to increase about four feet from the weir, and at two feet had increased about seven per cent.

The third experiment was made when the depth of the channel below the crest of the weir was 1.70 feet, and the depth on the weir was about 0.78 feet.

The current meter was held 0.40 feet above the bottom of the channel, 0.47 feet below the surface, and at mid-depth. The velocities near the bottom began to diminish about six or seven feet from the weir; at three feet they had decreased about seven per cent., and two feet from the weir about eleven per cent.

The velocities near the surface began to increase about five feet from the weir; at three feet they had increased about five per cent.; two feet from the weir about sixteen per cent., and were increasing very rapidly. The mid-depth velocities remained constant to within two feet of the weir.

6th. Motions of the water on the up-stream side of the weir.

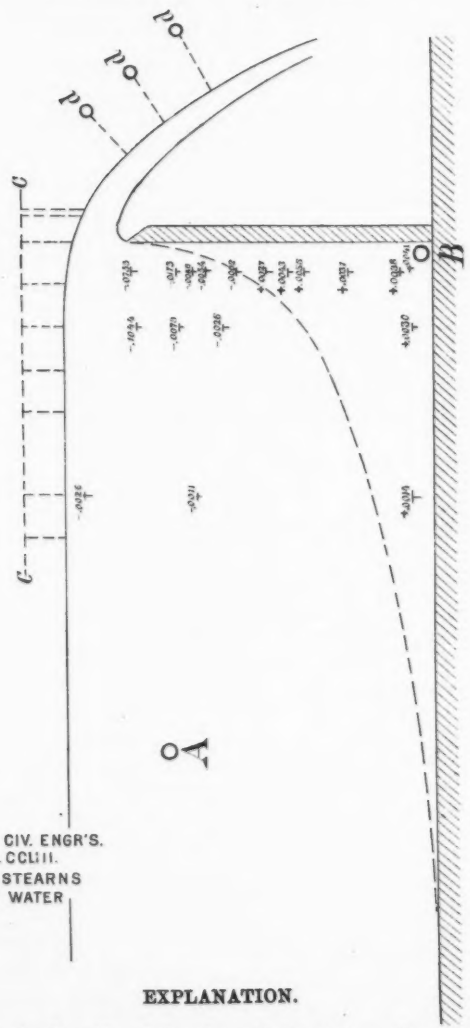
A small weir was constructed, and one side of the channel was made of glass, so that the motions of the water could be seen. The principal dimensions were: Width of channel and length of weir, each one foot; depth of channel below crest of weir, one foot; length of channel of approach, eight feet. A false bottom was provided to reduce the depth of the channel below the crest of the weir to 0.50 feet. By mixing partially water-logged sawdust with the water, the motions could be seen quite plainly, and the following observations were made:

a. An angle* existed between the face of the weir and the bottom of the channel, where the water had no continuous motion toward the weir.

* This angle should not be confounded with the angle of pressure mentioned on page 24, the latter being more extensive.

PLATE IV.

DIAGRAM SHOWING A COMPARISON BETWEEN HEADS TAKEN SIX FEET FROM THE WEIR AND AT VARIOUS OTHER PLACES.



EXPLANATION.

A is the orifice in the side of the channel 6 feet from the weir.
B is the perforated pipe at the bottom of the channel near the weir.

The small figures show the differences between heads at the points indicated, and at A; the head at A being the standard. The curve shown with a broken line is the upper limit of the angle where the head is greater than at A.

b. Water entered the angle near the centre of the channel, reached a point near the weir, turned horizontally toward the side of the channel, then passed up-stream, and rising at the same time with a spiral motion it finally went over the weir.

c. At the side of the channel the angle extended up-stream a distance equal to the height of the weir above the bottom of the channel. This rule appeared to hold good when the depth on the weir was changed from 0.05 to 0.50 feet, and with the bottom of the channel either 0.50, or 1.00 foot, below the crest of the weir.

The water within this angle was much agitated, and the prevailing motions hard to determine.

At a weir with end contraction, there is an angle at the side of the channel where the water has no continuous motion toward the weir, thus presenting a case somewhat similar to that just described.

Some observations of an angle of this kind are given in Table XII.

TABLE XII.

Depth of Bottom of Channel below Weir.	Depth on Weir, taken 6 feet from the Weir.	Distance from the side of the Channel to the end of the Weir.	Mean Velocity of Approach.	Length of the Angle along the side of the Channel.
3.56	0.9317	1.69	0.433	1.7
3.56	0.5765	1.69	0.230	2.0
1.00	0.5647	1.69	0 608	1.6
0.50	0.5574	1.69	0.901	2 6

In this case the length of the angle varies, though the distance from the side of the channel to the end of the weir is constant, but the cause of the variation is not apparent.

7th. Effect of velocity of approach upon the form of the surface curve of the sheet passing the weir.

The relative positions of the measurements taken to determine the changes in the form of the sheet, are shown on Plate IV.

The observations consisted of a measurement of the head at the orifice *A;* of vertical measurements from the line *c*, *c*, where shown by the

dotted lines; and of measurements from the points *d*, *d*, *d*, normal to the surface of the sheet.

Any one series of observations gave the form of the surface of the sheet; and successive observations with different velocities—the volume being constant—showed the effect of the change in velocity. The additional apparatus provided for taking the measurements is described below.

The horizontal line *c*, *c*, was the top of a steel straight-edge, securely fastened to a plank, 1.5 feet from and parallel with the north side of the channel. A spirit-level, resting on a ground-brass base and having the same degree of accuracy as those attached to leveling instruments, was used to determine when the straight-edge was level. The vertical measurements to the surface of the water were taken with the point-gauge mentioned on page 35. This gauge was attached to a frame having a triangular base, at each corner of which was a brass plate, upon which the apparatus rested. When the base was level the gauge was vertical. The two front corners rested on the straight-edge which made the gauge vertical in one direction, and the back corner rested upon a wedge by which it could be raised or lowered, until a spirit-level showed it to be vertical in the other direction. The points *d*, *d*, *d*, were nails. The measurements from them were taken with a small wooden rod having a square end, and may be in error as much as 0.01 feet. The positions of these nails were located by horizontal and vertical distances from the edge of the weir, as given in the following table:

No. of Nail.	Horizontal Distance from Edge of Weir to Nail.	Vertical Distance from Edge of Weir to Nail.
No. 1	1.545 ft. down-stream.	0.503 ft. upward.
" 2	2.01 " "	0.01 " "
" 3	2.52 " "	0.553 " downward.

While measurements were being taken with the point-gauge, the usual measurements were being taken with the hook-gauges, which furnished the data for making corrections for the minute variations, which occurred from time to time in the depth on the weir.

The measurements with the point-gauge were single observations—not the average of several, as in the other cases—and those taken near

the weir, where the surface of the water was much inclined, were liable to additional error on account of the difficulty in measuring exactly the distance, up-stream or down-stream, from the weir to the point where the measurement was taken.

Owing to the inaccuracies due to causes just mentioned, a tabulation of the measurements as taken did not show the general result clearly; therefore, in preparing a table (Table XIII.) to accompany this paper, each measurement has been given a value which, from a study of all the measurements, seemed most nearly correct. Where the measurements have been changed, their original value is shown in small figures to indicate to what extent the results of observation have been departed from.

An examination of the columns of differences shows that velocity of approach has less effect upon the depths nearly over the weir than farther up-stream, or, to express the case differently, velocity of approach diminishes the inclination of the water surface above the weir. To illustrate, take the extreme case given in the table:

In Exp. 1, the fall of the surface from six feet up-stream to 0.38 feet down-stream from the weir, was $0{\cdot}6143 - 0.3845 = 0.2298$ feet.

In Exp. 5, the corresponding figures are $0.5635 - 0.3660 = 0.1975$ feet. In the latter experiment the fall was 0.0323 feet less than in the former, or a very large proportion of the difference between the theoretical heads due to the velocities; this difference being $0.0363 - 0.0024 = 0.0339$ feet.

The measurements from the nails indicate but little variation of the surface of the sheet at the points considered.

The ratios of the depths measured directly over the edge of the weir, to the depths measured above the origin of the curve of the surface, are as follows:

	Ratios.	Mean Velocity of Approach.
Experiment 1.....	0.852	0.389
" 2.....	0.854	0.506
" 3.....	0.856	0.705
" 4.....	0.862	1.022
" 5.....	0.882	1.529

An experiment made by J. B. Francis, with a small velocity of approach, gives 0.853.

Conclusions.

From the experiments before described, some inferences may be drawn in regard to the manner in which a weir should be constructed, in order to reduce to a minimum the error made in calculating the velocity of approach.

The experiments show that, at a certain distance from the weir, the bottom velocities are retarded, and the angle of pressure, which is produced by this retardation, begins at the same point. Since the mean velocity in any cross-section of a uniform channel is always the same, it follows that the retardation of the bottom layers must be compensated for by a more rapid motion in the rest of the mass.

This acceleration is due, at least in part, to a loss of head at the surface.

It thus appears that a weir affects the flow in its channel as far up-stream as a certain point at which both the angle of pressure and the surface curve of the sheet begin. The experiments did not show that the acceleration at the surface began as far up-stream as the retardation at the bottom, but it is probably because the acceleration, acting upon a larger mass, was smaller and not appreciable so near its origin as the retardation at the bottom. This view has been taken by Boileau,* who says: "I would not venture to affirm that the point where the ascending motion of the liquid particles begins, corresponds vertically to the origin of the surface curve, although that circumstance seemed to me to have taken place."

The cause of the variation of the distance up-stream, at which this double change of velocity takes place, is not shown definitely by the experiments; but it is deemed reasonable and fairly in accordance with them to assume that the distance varies with the height of the weir above the bottom of the channel, and that it is about $2\frac{1}{2}$ times this height.

It is essential for the accurate application of the formulæ given, that the apparatus be so constructed as not to prevent the flow from assuming its normal condition for the distance from the weir given above.

Abnormal conditions of flow may be caused by placing screens too near the weir, by irregular velocities, by the lack of straightness and uniformity of the channel section, and by other causes.

* Traité de la mesure des Eaux Courantes, by P. Boileau. Paris, 1854, p. 57.

TABLE XIII.

MEASUREMENTS TO DETERMINE THE FORM OF THE SURFACE CURVE OF THE SHEET PASSING THE WEIR. MARCH 19, 1878.

1	2	3	4	5	6	7	8	9	10	
	EXPERIMENT No. 1.	Difference between Depths in Experiments 1 and 2.	EXPERIMENT No. 2.	Difference between Depths in Experiments 2 and 3.	EXPERIMENT No. 3.	Difference between Depths in Experiments 3 and 4.	EXPERIMENT No. 4.	Difference between Depths in Experiments 4 and 5.	EXPERIMENT No. 5.	
DISTANCE FROM THE WEIR.	Depth of Channel 3.56 Mean Vel. of App. 0.389 Theoretical Head due to Velocity 0.0024		Depth of Channel 2.60 Mean Vel. of App. 0.506 Theoretical Head due to Velocity 0.0040		Depth of Channel 1.70 Mean Vel. of App. 0.705 Theoretical Head due to Velocity 0.0077		Depth of Channel 1.00 Mean Vel. of App. 1.022 Theoretical Head due to Velocity 0.0162		Depth of Channel 0.50 Mean Vel. of App. 1.529 Theoretical Head due to Velocity 0.0363	
	Depth on the Weir.		Depth on the Weir.		Depth on the Weir.		Depth on the Weir.		Depth on the Weir.	
Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	
6.00 up-stream.	0.6143	0.0023	0.6120	0.0061	0.6059	0.0155	0.5904	0.0269	0.5635	
	0.6142		0.6117		0.6060		0.5874			
3.50 "	0.6140	0.0022	0.6118	0.0059	0.6059	0.0155	0.5904	0.0269	0.5635	
	0.6136		0.6126		0.6061				0.5631	
3.00 "	0.6135	0.0020	0.6115	0.0058	0.6057	0.0154	0.5903	0.0269	0.5634	
	0.6084		0.6102		0.6039				0.5621	
2.00 "	0.6114	0.0018	0.6096	0.0056	0.6040	0.0146	0.5894	0.0263	0.5631	
	0.6083		0.6066		0.6013		0.5882		0.5616	
1.50 "	0.6085	0.0017	0.6068	0.0053	0.6015	0.0136	0.5879	0.0253	0.5626	
					0.5939		0.5833		0.5596	
1.00 "	0.6017	0.0015	0.6002	0.0051	0.5951	0.0124	0.5827	0.0232	0.5595	
			0.5827				0.5671		0.5474	
0.50 "	0.5847	0.0012	0.5835	0.0047	0.5788	0.0110	0.5678	0.0190	0.5488	
	0.5226				0.5182		0.5097		0.5003	
0.00 "	0.5233	0.0007	0.5226	0.0041	0.5185	0.0094	0.5091	0.0123	0.4968	
	0.4251		0.4261		0.4212		0.4089		0.3914	
0.30 down-stream.	0.4253	0.0004	0.4249	0.0036	0.4213	0.0081	0.4132	0.0081	0.4051	
			0.3844		0.3786		0.3677		0.3739	
0.38 "	0.3845	0.0003	0.3842	0.0033	0.3809	0.0077	0.3732	0.0072	0.3660	
No. of Nail.	Distance to Sheet.		Distance to Sheet.		Distance to Sheet.		Distance to Sheet.		Distance to Sheet.	
No. 1.	0.875		0.870		0.860		0.865		0 873	
No. 2.	0.970		0.965		0.975		0.970		0.980	
No. 3.	1.140		1.135		1.140		1.140		1.150	

Corrected for loss of Head due to friction.

The experiments furnish also some data with regard to the best place for taking the head on the weir. The head, if measured outside of the angle of pressure, should be taken far enough up-stream from the weir to represent the height of the water surface above the beginning of the surface curvature, *i. e.*, at a distance from the weir equal to 2½ times its height above the bottom of the channel. So great a distance from the weir may not be necessary with deep channels; little harm, however, can result from taking the head too far from the weir if the channel is uniform; the amount of error being only the loss of head due to friction.

To give an idea of the amount of this loss, the inclination of the water surface, in two of the experiments, has been calculated by Darcy and Bazin's formula for flow through channels lined with planed boards, and the results are given in the following table:

Depth of Channel below Crest of Weir.	Depth on Weir.	Mean Velocity.	Inclination of Water Surface.
Feet.	Feet.	Feet.	
3.56	0.9443	0.693	0.001 in 67 feet.
1.00	0.8854	1.656	0.001 " 8 "

If it became necessary to take the head somewhat nearer the weir, it would be well to place the orifice at mid-depth, where it would be intermediate between the loss of head at the surface and the increase of head at the bottom, and where the measurements, made with the current meter (see pages 41 and 42), showed the velocities to be quite uniform to within a comparatively short distance from the weir.

When the head is measured within the angle of pressure, it is thought best to take it with a perforated tube, as was done in these experiments.

The excess of head within the angle is dependent upon the velocity of approach, and the head will be greatest where the velocity is greatest, *i. e.*, generally at the middle of the channel. Although this is theoretically true, and may cause a sensible difference of heads in some cases, it is not entirely in accordance with the statement made by Boileau*, that the liquid column (in his gauging tube) rose to the same mean height at all points of the width of the channel, except when the tube was only at

* Traité de la mesure des Eaux Courantes, p. 51.

a few millimeters from the vertical wall of the channel, where the liquid column was depressed.

The experiments recorded on Plate IV. indicate that the excess of head at the bottom is less than at a point 1.5 feet above it.

Of the two places used for taking the heads, that outside the angle of increased pressure furnished the most reliable results. This was very noticeable when using a weir with end contractions, and also in the case of a weir without end contractions, when the velocities in the channel of approach were irregular.

In addition to the data already given, some measurements taken by other experimenters and by ourselves, with other apparatus, show a remarkable variation in the excess of head within the angle, caused, apparently, in several instances, by the proximity of the screens to the weir.

One series of measurements was taken at the weir shown on Plate III. and described on pages 61 to 66. In this the height of the weir above the bottom of the channel was 6.55 feet, and the screens were 7.45 feet from the weir.

The excess of head within the angle, as compared with that taken near the surface 6 feet from the weir, varied from 2.36 to 2.64 h, h being the theoretical head due to the mean velocity of approach. Another case is selected from a series of measurements in which the water, after passing the large weir (Plate III.), which had been previously tested, flowed over two weirs placed at the same level in adjoining bays in one of the structures on the line of the conduit. It had been ascertained previously that the amount of water furnished by the conduit itself, through leakage or other causes, between the two points of observation, was not sufficient to cause an error which would practically affect the result.

The total length of the two weirs was 13.56 feet. End contractions were suppressed. The height of the weirs was 2.76 feet. The distance from the weirs to the screens was 3.0 feet only. The head was taken by pipes placed on the bottom of the channels and ending at their respective centres near the weirs. These pipes were connected together, and a single pipe led from the junction to the gauge. The head above the surface curvature could not be measured, but the value which it would have had, if the screens had been sufficiently distant from the weir, can be calculated since the volume passing the weir is known.

The results were nearly the same for all depths on the weir; they are given for one case:

Observed depth on the weir, as taken in the angle	1.50 ft.
Volume passing the weir per lineal foot	5.854 cu. ft. per sec.
Depth on the weir above the surface curvature, calculated from the formula for discharge, $Q = 3.31\ LH^{\frac{3}{2}} + 0.007\ L$ (see p. 82); after correcting for velocity of approach by the formula $C = 1.39\ h$	1.419 ft.
Mean velocity of approach	1.40 ft. per sec.
Theoretical head due to mean velocity (h)	0.0305. ft.
Excess of head in angle	2.66 h.

Mr. Francis has recorded two series of experiments, in some of which heads taken within the angle near the weir are compared with heads taken at the bottom of the channel 12 feet from the weir.

The latter heads are taken at points where the excess of pressure, if any exists, is small, and consequently the comparison shows the whole excess of head close to the weir, or a quantity a little smaller.

The first series* was made at the Lower Locks, in November, 1852. During the experiments of November 7th the weir was without end contractions. The channel was about 20 feet long, and unobstructed.

The bottom of the channel sloped gradually upwards toward the weir, which was 4.60 feet high. The velocity of approach was so small that minute errors in measurement would cause a considerable change in the co-efficients. In two cases, where the comparison could be made, the depths on the weir were 0.81 and 0.98 feet, and the excess of head in the angle, respectively, 0.62 and 0.46 h, h being the theoretical head due to the mean velocity 6 feet from the weir. The record of the experiments of November 3d and 4th did not furnish sufficient data for comparison in this case.

The second series† was made at the Tremont weir in the fall of 1856. The water flowed over two weirs with end contractions. The total length of the weirs was about 80 feet. The width of the channel was 88 feet. The height of the weir was 6 feet, and the screens were about 15 feet from the weir. In one experiment the depth on the weir was 1.44

* Lowell Hydraulic Experiments, 3d Ed., p. 142.
† Lowell Hydraulic Experiments, 3d Ed., p. 183.

feet, the theoretical head due to the mean velocity of approach was 0.0076 feet, and the excess of head near the weir was 0.0124 feet, equal to 1.63 h.

Three series of experiments on the same subject are recorded by Boileau.* In all of them the head in the angle of increased pressure was taken with a vertical tube standing against the weir; the other measurement was made to the surface of the water above the point where the gradual depression of the surface commences. In the first series the height of the weir was 1.076 feet, the depth of water over the weir varied from 0.419 to 0.759 feet, and the excess of head at the weir was from 0.1 h to 0.64 h.

In the second series the height of the weir was 1.998 feet, the width was 0.98 foot, the depth over the weir varied from 0.817 to 1.604 foot. The first two experiments showed no difference; six others showed an excess of head near the weir of 0.15 h to 0.33 h.

The third series was made on a weir 0.676 foot high and 2.953 feet wide, with depths over the weir varying from 0.144 to 0.954 foot. The first four experiments showed a very large excess, and may be unreliable, owing to the small quantities involved; the remaining six showed an excess varying from 0.47 h to 0.60 h.

Mention is made of some experiments with a weir 2.66 feet high, in which, instead of an excess of head within the angle, a difference was found in the other direction.

The experiments cited show that in some instances the excess of head within the angle of increased pressure is several times larger than the theoretical head due to the velocity of approach; this result is, apparently, caused, in some cases, by the proximity of screens to the weir, or by irregular velocities; but why it is so we are unable to explain.

How near the weir screens may be placed without affecting the accuracy of measurement is not definitely known. It seems evident that they should be placed above the point where the flow in the channel is first affected by the weir, *i. e.*, at a distance above the weir greater than $2\frac{1}{2}$ times its height.

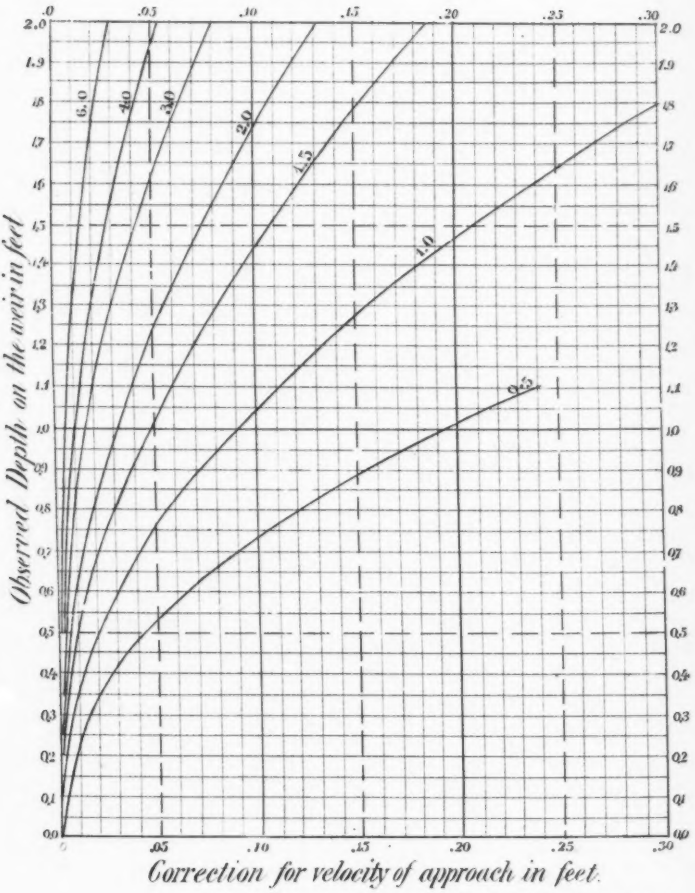
Two or three feet more than this distance would probably be ample, since the experiments on page 37 showed that there was little change in the relative velocities in the channel from a point 1.7 feet below the screens to a point where the effect of the weir was first felt.

* Traité de la mesure des Eaux Courantes, by P. Boileau. Paris, 1854, pp. 58, 60, 61.

PLATE V.

DIAGRAM SHOWING THE CORRECTIONS FOR VELOCITY OF APPROACH TO BE ADDED TO THE DEPTHS ON WEIRS WITHOUT END CONTRACTIONS.

NOTE.—The figures on the curves show depths of channel below crest of weir.



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ON FLOW OF WATER

During one experiment (see page 38), in which the screens were placed 7.3 feet from a weir 3.56 feet high, the discharge was affected little, if any. In this case the distance was less than $2\frac{1}{2}$ times the height of the weir, and it may be said to corroborate the statement given above; yet, with the present limited knowledge of the subject, it would be safer to place the screens as far from the weir as practicable.

The circumstances which may influence the effect of velocity of approach on the flow over a weir are so numerous and so varied in their nature, that the importance of using an apparatus of the best and simplest form for the determination of reliable data is obvious.

For the same reasons it seems equally obvious that, in order to render applicable the formula given, the conditions which existed during the experiments must be observed. The more important ones are:

1st. A sufficient length of channel between the screens and the weir.

2d. A straight channel of uniform section.

3d. The head, if taken without the angle of pressure, should be taken at such a place and in such a manner that it will represent correctly the height of the surface of the water above the curvature of the sheet; or, if taken within the angle, it should be taken at the bottom of the channel close to the weir.

In practice the calculation of the correction for velocity of approach is attended with considerable labor; but where the results of many observations are to be obtained, the labor may be greatly reduced by plotting for any given weir, with or without end contractions, a curve, similar to those shown on the accompanying diagram (Plate V.), which gives to the natural scale the correction to be added to the observed depth on a weir without end contractions for various depths of channel.

The quantities involved being small, a diagram can be made to a scale sufficiently large to furnish the results with any required degree of accuracy.

EXPERIMENTS ON THE FLOW OF WATER OVER SHARP-CRESTED WEIRS.

These experiments were made on two different weirs. The experiments made on the first, which was 5 feet long, with depths varying from 0.07 to 0.83 feet, took place in March, 1877.

Those made on the second weir, 19 feet long, with depths varying from 0.47 to 1.63 feet, took place in November and December, 1879.

In carrying out these experiments the information given by Mr. J. B. Francis has been an invaluable guide for the conduct of the observations and for the arrangement of the apparatus. The first object of the smaller weir was to ascertain whether the co-efficient given by him is equally applicable when the depths of water over the weir are smaller than those which he had experimented upon, and if not, to find a formula which could cover all depths within experimental limits. The apparatus was also used for measuring the flowing capacity of the conduit for small volumes.

The larger weir was constructed for measuring the flowing capacity of the conduit for larger volumes; and as the conditions of flow over the weir for large depths were different from those recommended by Mr. Francis, several experiments were made to test it. In both cases the flow of water was actually measured in a basin below the weir.

EXPERIMENTS ON WEIR FIVE FEET LONG.

In 1877, at the time of the observations, the Gate-House at Farm Pond was not completed, and a bulkhead at the entrance of the conduit retained the water of Farm Pond.

The apparatus was entirely within the conduit, and is shown on Plate I. Fig. 1 is a plan in which the upper arch of the conduit is supposed to have been removed. Fig. 2 is a longitudinal section. Fig. 3 is an up-stream elevation of the weir, and shows also a section of the channel

of approach and of the conduit. Fig. 4 shows a section of the weir on a larger scale. The same letters refer to the same parts of the apparatus in each of the figures. *B*, *B*, *B*, *B*, are four gates which controlled the flow of water through openings in the bulkhead at the entrance of the conduit; these gates could be raised to admit any desired quantity of water into the basin above the weir, and could be clamped at any height. *C* is a box into which the water entering through the orifices fell. *D* is a plank bulkhead having a few openings in it. *E* is a screen. *F*, *F*, *F*, *F*, are floating planks provided to reduce the oscillations of the surface. *G*, *G*, are the sides of the channel of approach, which was about 12 feet long and 5 feet wide. The bottom of the channel was curved, as shown in Fig. 3.

The mean depth of the channel below the crest of the weir was 3.17 feet. The sides of the channel were made of matched boards, and a portion of them (shown at *K*) near and at the weir were made of carefully planed boards, which extended a short distance below the weir. This disposition prevented both the end contraction and the expansion of the sheet after passing the weir; and it may be said that the weir used in such conditions represented fairly a section of a weir of infinite length. *I* is the weir which is also shown on a larger scale in Fig. 4. The water flowed over a nickel-plated steel straight-edge 0.0066 feet thick. The up-stream edge was sharp, and during the experiments the water never touched any portion of the weir after passing this edge. *N* is a gate by which the weir basin could be emptied.

The depth of water on the weir was measured with a hook-gauge firmly fastened to a post, *O*, placed 6 feet below the weir. The hook-gauge pail, placed on a movable shelf, was connected by means of a rubber pipe with an orifice in the board *H*, 6 feet above the weir. *L* and *M* are two gates 5 feet long, either of which could be used to close the weir opening. The gate *M* (see Fig. 9) was held in place, when closed, by two bolts. The gate *L*, when closed, was held in place by the pressure of the water. *P* is a dam, 22 feet below the weir, having an opening which could be closed with stop-planks. The basin between this dam and the weir was used to measure the discharge over the latter when the depths of water flowing over it were small, and is designated as the "small basin." A similar dam was built 367 feet below the weir, and the basin between this dam and the weir goes by the name of the "large basin." Below each of these dams a smaller one (shown at *Q*) was built,

which formed a small pool from which water leaking through the stop-planks was bailed back into the measuring basin. Five screens were placed at intervals across the "large basin" to prevent oscillations from one end to the other. The height of the water in the measuring basin was taken with a hook-gauge attached to the up-stream side of the first dam below the weir.

In making experiments of this kind the depth on the weir must be maintained nearly uniform from the beginning. This result is usually attained by allowing the water to flow over the weir until the depth remains nearly constant, and by diverting it from the measuring basin until the beginning of an experiment. In the present case the water passing the weir could not be diverted, and it was therefore necessary to allow no water to pass the weir except during the experiments, and to start the flow with the depth on the weir which was to be maintained. To attain this result the opening of the gates in the bulkhead necessary to give the required depth on the weir was ascertained by a preliminary trial; stops were then fastened above the gates so that they could be suddenly opened to the same width. At the beginning of the experiment the weir opening was closed by the gate on the down-stream side, and sufficient water was admitted to raise the level in the basin above the weir very gradually until it reached the desired depth on the weir, then, at a signal, the gates at the weir and at the bulkhead were suddenly opened. The gate at the weir (Plate I., Fig. 9) was so arranged that a single pull of the rope drew the bolts and opened the gate at the same time; the movement of the gate was further aided by the water pressure. To end an experiment the gate on the up-stream side of the weir and the gates in the bulkhead were closed nearly simultaneously.

The duration of the experiment was measured with a stop-watch which recorded fifths of seconds. The watch was not a very accurate time-keeper, but was frequently compared with a reliable watch to ascertain its rate. The hook-gauge was read four times in the first minute, then twice a minute until the flow became constant, then every minute until the end.

At the beginning of an experiment there was water in sufficient quantity in the measuring basin to cover the invert of the conduit, and during the experiment it was allowed to rise about two feet, so that the volume of water measured had a section very nearly rectangular, limited at top and bottom by level surfaces.

The formula adopted for calculating the results of the experiments was of the well-known form $Q = c\ L\ H_2^{\frac{3}{2}}$, in which

Q is the quantity in cubic feet per second,

c a co-efficient,

L length of weir,

H depth on weir corrected for velocity of approach.

The value of c was calculated from each experiment and was found to vary with the depth on the weir.

A formula, containing only constant co-efficients, yet representing the results, was next sought, and found to be of the form

$Q = c\ L\ H^{\frac{3}{2}} + a\ L$, in which c and a are constant quantities; to represent this series of experiments the formula becomes

$Q = 3.33\ L\ H^{\frac{3}{2}} + 0.0065\ L$.

Table XIV. shows the results of all the experiments, also a comparison with the results obtained by the above formula.

Each experiment is also plotted on Plate VI. in the vicinity of the curve marked "A," which represents the formula. The circles and crosses represent respectively the "large" and "small" basin experiments. A query-mark has been placed beside those which are imperfect, as shown in the column of remarks (Table XIV).

The proportional differences resulting from the comparison between the formula and the experiments are given in column 14 of the table. Upon examination it shows many irregularities, especially when the water was measured in the "small basin." To sum up the results given, we find 16 "large basin" experiments with depths on the weir varying from 0.8312 to 0.2296 feet; of these one (No. 7) is shown by the result to be of no value; three others were imperfect, as may be seen by referring to the columns of remarks, and are given only for the purpose of presenting a complete record; their average difference from the formula is 0.0041; of the remaining twelve the largest difference is 0.0079; the next largest, 0.0025, and in no other case does it exceed 0.0010.

The "small basin" experiments, with depths on the weir ranging from 0.2193 to 0.0735 feet, are 15 in number; of these 3 were imperfect and showed an average difference of 2½ per cent.; the remainder showed an average difference of 1½ per cent.

A careful study of the experiments seemed to show that the irregularities were due to causes which might affect some without affecting the whole, and which were as likely to produce errors in one direction as in

the other; the conclusion was therefore reached that 10 of the "large basin" experiments contained no errors of consequence, and that the errors in the "small basin" experiments were not beyond the limits indicated by the column of differences. The results do not show a very close accordance with other results of our own, nor with those of other experimenters, and for this reason, as well as for justifying the conclusions given above, it may not be out of place to discuss with some detail the circumstances of the experiments.

The principal errors which may have occurred are as follows:

1st. Errors in physical measurements, such as the measurement of the depth on the weir, of its length, of the duration of the experiment and of the size of the measuring basin.

2d. Errors due to the conditions existing during the experiments, such as the velocity of approach, the friction at the ends of the weir imperfections in the construction of the apparatus, etc.

3d. Errors due to leakages into and out of the gauging basin.

Before proceeding with details it is well to state that, owing to the lack of room at the place where the experiments were made, it was necessary to change the assistants frequently to relieve them from their cramped positions. This, by preventing a proper organization of the force employed, was the principal occasion of the large number of imperfect experiments and a cause of some of the irregularities in others; most of the irregularities, however, were caused by errors in the measurement of depth on the weir, which will be considered.

Depth on the Weir.

Owing to the manner in which the experiments were started, the level of the water in the basin above the weir fluctuated to a greater or less extent for a short time at the beginning of an experiment.

In most of the experiments this fluctuation did not exceed a few thousandths of a foot; in the remainder it was larger, reaching 0.049 of a foot in the extreme case (Exp. No. 9). Such initial fluctuations could not be definitely measured with the apparatus used, as it was impracticable to take measurements oftener than once in fifteen seconds, and for the more important reason that, owing to the inertia of the water in the connecting pipe, fluctuations in the basin above the weir were recorded but imperfectly, and after an interval of time, in the gauge pail. An error of any given amount, occurring in this way, would affect the

TABLE XIV.

EXPERIMENTS ON THE FLOW OF WATER OVER A SHARP-CRESTED WEIR, WITHOUT END CONTRAC

DEPTH OF CHANNEL BELOW CREST OF WEIR, 3.17 FEET. TEMPERATURE OF

1	2	3	4	5	6	7	8	9	10	V
Number of the Experiment.	Date of the Experiment. —— 1877.	Observed Depth on Weir.	Mean Velocity of Approach.	Theoretical Head due to Mean Velocity of Approach. (h_1)	Correction for Velocity of Approach by Formula $C = 1.8\,h_1$.	Depth on the Weir Corrected for Velocity of Approach. (H)	Length of Weir. (L)	Total Volume of Water which Passed the Weir during the Experiment.	Duration of the Experiment.	of Pass per
		Feet.	Ft. per sec.	Feet.	Feet.	Feet.	Feet.	Cubic feet.	Seconds.	C. f
					"LARGE BASIN" EXPERIMENT					
1	March 24.	0.8198	0.639	0.0064	0.0114	0.8312	5.000	6712.7	526.5	1
2	" 24.	0.8121	0.627	0.0061	0.0110	0.8231	4.997	6755.2	*541.2	*1
3	" 24.	0.8118	0.627	0.0061	0.0110	0.8228	5.000	6391.5	512.7	1
4	" 24.	0.6761	0.490	0.0037	0.0067	0.6828	5.000	6579.2	697.7	
5	" 24.	0.6713	0.486	0.0037	0.0066	0.6779	4.997	6726.7	721.6	
6	" 26.	0.5203	0.344	0.0018	0.0033	0.5236	4.996	6296.5	992.9	
7	" 22.	0.4810	0.316	0.0016	0.0028	0.4838	4.999	5999.3	1040.5	
8	" 22.	0.4761	0.304	0.0014	0.0026	0.4787	4.999	6725.1	1212.4	
9	" 22.	0.4569	0.287	0.0013	0.0023	0.4592	4.999	6417.3	1234.3	
10	" 22.	0.3890	0.230	0.0008	0.0015	0.3905	4.999	6646.0	1623.5	
11	" 27.	0.3424	0.193	0.0006	0.0010	0.3434	4.994	6342.9	*1870.2	*
12	" 21.	0.3407	0.191	0.0006	0.0010	0.3417	4.994	6585.0	1963.3	
13	" 23.	0.3114	0.169	0.0004	0.0008	0.3122	4.998	6870.2	2340.4	
14	" 22.	0.2598	0.131	0.0003	0.0005	0.2603	4.999	6199.8	2765.9	
15	" 22.	0.2467	0.122	0.0002	0.0004	0.2471	4.999	6247.1	3006.3	
16	" 23.	0.2293	0.110	0.0002	0.0003	0.2296	4.998	6532.0	*3481.2	*
					"SMALL BASIN" EXPERIMEN					
17	March 21.	0.2190	0.103	0.0002	0.0003	0.2193	5.000	348.6	199.5	1
18	" 21.	0.2182	0.102	0.0002	0.0003	0.2185	5.000	348.0	202.2	1
19	" 21.	0.2176	0.105	0.0002	0.0003	0.2179	5.000	370.3	207.6	1
20	" 23.	0.1650	0.070	0.0001	0.0001	0.1651	4.998	363.1	310.2	1
21	April 2.	0.1627	0.068	0.0001	0.0001	0.1628	4.995	352.6	310.2	1
22	" 2.	0.1444	0.057	0.0001	0.0001	0.1445	4.995	369.4	390.1	0
23	March 29.	0.1235	0.045	0.0000	0.0001	0.1236	4.998	345.6	461.1	0
24	" 29.	0.1225	0.046	0.0000	0.0001	0.1226	4.998	338.9	450.3	0
25	" 21.	0.1127	0.040	0.0000	0.0000	0.1127	5.000	371.7	564.0	0
26	" 21.	0.1125	0.038	0.0000	0.0000	0.1125	5.000	353.7	*561.2	*0
27	" 31.	0.1009	0.036	0.0000	0.0000	0.1009	4.996	351.2	600.8	0
28	" 31.	0.1008	0.036	0.0000	0.0000	0.1008	4.996	361.8	615.6	0
29	" 31.	0.0991	0.034	0.0000	0.0000	0.0991	4.996	363.5	661.2	0
30	" 31.	0.0746	0.023	0.0000	0.0000	0.0746	4.996	372.9	1021.1	0
31	" 31.	0.0735	0.023	0.0000	0.0000	0.0735	4.996	369.4	*990.7	*0

TABLE XIV.

WEIR, WITHOUT END CONTRACTION; MADE IN THE SUDBU

17 FEET. TEMPERATURE OF WATER VERY NEARLY CONS

8	9	10	11	12	13
Length of Weir. (L)	Total Volume of Water which Passed the Weir during the Experiment.	Duration of the Experiment.	Volume of Water Passing the Weir per Second. (Q)	Value of c in the Formula $Q = cLH^{\frac{3}{2}}$.	Volume of Water passing the Weir Calculated by the Formula $Q = 3.33\ LH^{\frac{3}{2}} - .0065\ L$.

"LARGE BASIN" EXPERIMENTS.

Feet.	Cubic feet.	Seconds.	C. ft. p. sec.		Cu. feet per se
5.000	6712.7	526.5	12.750	3.365	12.650
4.997	6755.2	*541.2	*12.482	*3.345	12.459
5.000	6391.5	512.7	12.466	3.340	12.459
5.000	6579.2	697.7	9.430	3.343	9.427
4.997	6726.7	721.6	9.322	3.342	9.320
4.996	6296.5	992.9	6.342	3.350	6.336
4.999	5999.3	1040.5	5.766	3.428	5.634
4.999	6725.1	1212.4	5.547	3.350	5.546
4.999	6417.3	1234.3	5.199	3.342	5.212
4.999	6646.0	1623.5	4.094	3.356	4.095
4.994	6342.9	*1870.2	*3.3916	*3.375	3.3790
4.994	6585.0	1963.3	3.3540	3.362	3.3542
4.998	6870.2	2340.4	2.9355	3.367	2.9358
4.999	6199.8	2765.9	2.2415	3.376	2.2432
4.999	6247.1	3006.3	2.0780	3.384	2.0772
4.998	6532.0	*3481.2	*1.8764	*3.412	1.8635

"SMALL BASIN" EXPERIMENTS.

Feet.	Cubic feet.	Seconds.	C. ft. p. sec.		Cu. feet per se
5.000	348.6	199.5	1.7474	3.403	1.7424
5.000	348.0	202.2	1.7211	3.370	1.7331
5.000	370.3	207.6	1.7837	3.507	1.7261
4.998	363.1	310.2	1.1705	3.491	1.1490
4.995	352.6	310.2	1.1367	3.464	1.1251
4.995	369.4	390.1	0.9469	3.451	0.9462
4.998	345.6	461.1	0.7495	3.451	0.7557
4.998	338.9	450.3	0.7526	3.508	0.7470
5.000	371.7	564.0	0.6590	3.484	0.6624
5.000	353.7	*561.2	*0.6303	*3.341	0.6608
4.996	351.2	600.8	0.5846	3.651	0.5657
4.996	361.8	615.6	0.5877	3.676	0.5649
4.996	363.5	661.2	0.5498	3.528	0.5515
4.996	372.9	1021.1	0.3652	3.588	0.3715
4.996	369.4	*990.7	*0.3729	*3.746	0.3640

SUDBURY CONDUIT IN MARCH AND APRIL, 1877.

CONSTANT AT 36° FAHRENHEIT.

13	14	15	16
lume of r passing e Weir ulated by Formula $3.33\ LH^{\frac{3}{2}} + .065\ L.$	Proportional Difference between Volume as Calculated and as Measured, using the Calculated Volume as the Basis.	Variation in the Depth on the Weir during the Experiment.	REMARKS.
eet per sec.		Feet.	
12.650	+ 0.0079	0.037	
12.459	* + 0.0018	0.045	Time not properly recorded.
12.459	+ 0.0006	0.040	
9.427	+ 0.0003	0.009	
9.320	+ 0.0002	0.035	
6.336	+ 0.0009	0.006	
5.634	+ 0.0234	0.001	The results show that this experiment is worthless.
5.546	+ 0.0002	0.005	
5.212	— 0.0025	0.049	
4.095	— 0.0002	0.004	
3.3790	* + 0.0037	0.003	The gate at the Weir opening was not closed for some seconds after the signal to end the experiment was given.
3.3542	— 0.0001	0.009	
2.9358	— 0.0001	0.005	
2.2432	— 0.0008	0.006	
2.0772	+ 0.0004	0.004	
1.8635	* - 0.0069	0.005	Time not properly recorded.
eet per sec.		Feet.	
1.7424	+ 0.0029	0.001	
1.7331	+ 0.0069	0.005	
1.7261	+ 0.0334	0.003	
1.1490	+ 0.0187	0.007	
1.1251	+ 0.0103	0.011	
0.9462	+ 0.0008	0.007	
0.7557	— 0.0082	0.003	
0.7470	+ 0.0075	0.003	
0.6624	— 0.0051	0.012	The gate at the Weir dropped and partially obstructed the flow for a short time before the experiment ended.
0.6608	*— 0.0461	0.001	Time not properly recorded.
0.5657	+ 0.0334	0.017	
0.5649	+ 0 0404	0.020	
0.5515	— 0.0031	0.044	
0.3715	— 0.0170	0.010	
0.3640	* + 0.0245	0.003	Time not properly recorded.

final results less, as the duration of the experiment and the depth on the weir increased, varying in a direct ratio in both cases. It has been thus found that the "small basin" experiments were affected on an average five times as much as those made in connection with the "large basin."

Aside from the errors referred to, the depths on the weir were very carefully taken; in fact, the necessity for accuracy was so apparent with the smallest depths on the weir that trials were made to ascertain the degree of accuracy attainable with the apparatus used. The comparative levels of the hook-gauge and of five different points of the weir crest were generally taken every two or three days with an apparatus similar to that described on page 63. On March 31, the day of the experiments with the smallest depths, two comparisons were made as given below; the observers at the gauge and weir changing places between the comparisons.

Point on Weir.	Reading of the Hook-gauge when the Water is Level with the Crest.		Average of First Observations.	Average of Second Observations.
	1st Comparison.	2d Comparison.		
No. 1.	0.0016	0.0019		
" 2.	0.0020	0.0020		
" 3.	0.0018	0.0019	0.00174	0.00182
" 4.	0 0018	0.0020		
" 5.	0.0015	0.0013	Difference, 0.00008	

In order to ascertain how exactly gauge measurements could be taken, several trials were made to test the accuracy with which the vernier could be read, and with which the point of the hook could be made to coincide with the surface of the water. In the former case three observers read the vernier,* while the position of the gauge remained unchanged. The average of these three readings was called the standard. From several trials it was found that, disregarding signs, the average difference from the standard was, with the three observers, 0.00004, 0.00007 and 0.00007 feet. The average personal error between two of the

* The vernier gave the readings of the gauge to thousandths of a foot; ten thousandths were estimated.

observers was 0.00012 feet. In the latter case the point of the hook was made to coincide as nearly as possible with the surface of water remaining at a constant level, and the very slight differences in setting the hook at successive trials were measured by the horizontal displacement of a beam of light reflected from a small mirror fastened to the screw of the gauge on a scale where the motion of the hook was magnified 3 000 times. Successive observations by the same observer showed an average difference of 0.000012 feet in height; similar observations with different observers, 0.000021 feet. Even these small quantities do not represent the limit of accuracy attainable, as the larger part of the differences is probably due to the imperfection of some of the arrangements for measuring the height.

Velocity of Approach.

It must be remembered that when these experiments were made, our investigations of this subject had not taken place.

Its effect was first computed by adding to the observed depth on the weir the head due to the mean velocity in the channel of approach. The co-efficients in the weir formula calculated upon this basis, instead of remaining constant, or decreasing continuously with increasing depths on the weir, as we were led by the results of experiments on record to suppose they should, showed a noticeable increase with the largest depths. A study of the experiments seemed to show no cause for this, except error in the correction for the effect of the velocity of approach, which appeared to be too small. In view of this fact, and because it was found to be warranted by the experiments of others, a correction of $1.8\,h$ was chosen as a basis for recalculating the experiments. The co-efficient 1.8 was selected in preference to any other, because it seemed to give the best results; moreover, it happened to be identical with the co-efficient deduced from Boileau's experiments.*

Our later experiments on velocity of approach furnish for this case a correction varying from 1.45 to 1.50 h. This difference may be due to the fact that the conditions (see page 47) relating to the manner of taking the head were not strictly complied with. The head was taken through a pipe ending at an auger hole in a planed board, 6 feet above the weir and 0.9 feet above the bottom. The board was placed 1.5 feet from and

* P. Boileau. Traité de la mesure des Eaux Courantes. Paris, 1854.

parallel with the side of the channel. Care was taken to prevent the end of the pipe from projecting beyond the board; but it should also be stated that the auger hole was about $\frac{1}{8}$ inch larger than the pipe.

There is hardly any doubt that the place where the head was taken was outside of the angle of pressure.

The differences in the final results occasioned by using the correction $1.8\,h$ instead of $1.45\,h$ or $1.50\,h$, as afterwards found, are not great; with the largest depth on the weir the variation is 0.4 of one per cent.; with 0.5 feet depth, 0.16 of one per cent.; with 0.3 feet depth, 0.06 of one per cent.; with the smaller depths it is inappreciable.

Length of Weir.

The length of the weir was measured every two or three days with a wooden rod, which was compared each time with a standard, and there can be no error of importance in the measurement. The effective length of the weir may have been less, owing to friction at the sides of the channel; if so, the co-efficients should be larger than those given.

Duration of the Experiment.

Errors in observing the beginning and end of the experiments would not probably exceed an average of one-half second, though in some experiments the total error in the duration may equal one or two seconds; an error of one second would affect the final results from $\frac{1}{2}$ to $\frac{1}{3}\frac{1}{5}$ of one per cent.

Size of the Measuring Basin.

The widths of the conduit were taken every 5 feet in its length for the "large basin," and every 3 feet for the "small basin;" in each case they were taken every three inches in height. The capacity between the "high" and "low" water levels was, for the "large basin," 6 403 cubic feet, and for the "small basin," 359.5 cubic feet. Graphical measurements of the cross-section of the conduit, previously taken every 25 feet in its length, were made use of for checking these capacities. The variation in the former case was $\frac{1}{1450}$, and in the latter case $\frac{1}{450}$.

In all cases proper deductions were made for posts, and proper corrections for any variation in the level of the water from the standard "high" and "low" water levels in the basin. The heights of the water

were taken with a hook-gauge, but, owing to oscillations of the surface, when at the high level, they were not taken so accurately as the depths on the weir. In the "small basin," however, the oscillations ceased after a short time ; in the "large basin" they were largely prevented by the five screens across the conduit, and as the stop-planks between the two portions of the basin were also put in, leaving but a small opening for the water, the amplitude of the oscillations at the gauge was reduced to about 0.006 feet. Eight or ten readings were generally taken, and the average could hardly be in error more than 0.001 foot, which would produce an error in the final results of $\frac{1}{10}$ of one per cent.

Leakage into or out of the Measuring Basins.

The basins were below the level of the water in the ground through which the conduit is built, and consequently received some water from percolation through the brick work. This was measured in the "large basin" when the water was at the "low" level, and was found at three different times to be 0.20, 0.21 and 0.22 cubic feet per minute. At the "high" level it would have been a little less, but no account was taken of this fact, since the total percolation in the longest experiment amounted to only $\frac{1}{600}$ of the capacity of the basin. At the end of a few of the "small basin" experiments there was a little leakage past the weir, which was taken account of by noting the rate at which the water rose in the basin.

Water leaking through the stop-planks, forming the lower ends of the basins, was collected into the small pools made below, and was bailed back into the basins ; in a few of the "small basin" experiments a little of that water was lost.

In conclusion, it may be stated, that, aside from what has already been described, no imperfections, which would appreciably influence the results, were known to exist, either in the apparatus used or in the methods employed.

EXPERIMENTS ON THE WEIR 19 FEET LONG.

These experiments were made in 1879. The weir was placed in the gate-house at Farm Pond. Plate III. shows the gate-house and the apparatus contained in it.

Fig. 1 is a longitudinal section. Fig. 2 is a general plan. Fig. 6 is a partial elevation of the weir looking up-stream.

A glance at the plan discloses the fact that the distance from the screens to the weir is much less than we have recommended in previous pages, and is not warranted by any previous experience.

As, however, the weir was put up to measure with accuracy the flowing capacity of the conduit with large volumes of water, the experiments were made to establish a reliable formula applicable to this weir. With large flows it was expected that the results might vary considerably from those obtained under normal conditions, but with smaller depths the flow of the water to the weir appeared so regular, and the velocity was so small, that it was not thought that the abnormal conditions would have any appreciable effect.

From Farm Pond the water entered the gate-house through the openings, A, A, in the masonry, and passed through wire screens at B. The two iron gates, C, C, were used for regulating the amount of water admitted to the weir, but their motion being necessarily slow, the swinging gates, F, F, were resorted to for the purpose of admitting the water suddenly at the beginning of each experiment.

These gates, made of planks, were supported from above by arms allowing them to swing backward and forward.

When they were closed against the masonry, as shown in the lower chamber (Fig. 2), they covered the openings D, D.

In this position they could be tightly clamped, and would hold back the water in the pond when the iron gates C, C, were opened. The fastenings of the swinging gates were so arranged that the withdrawal of bolts released them, and allowed them to open. By means of levers and suitable connections, the bolts of both gates could be withdrawn simultaneously by one person standing on the floor of the gate-house.

When they were open, as shown in Fig. 1 and in the upper chamber Fig. 2, they interrupted the swift current coming from the gate openings above, and the water passing over, under and either side of them, and

changing its direction several times, reached the openings, G,G, with a comparatively uniform velocity.

The iron gates G',G', in front of the openings G,G, were kept open all of the time.

Two screens, H,H, 4 feet wide and 6 feet high, were placed directly in front of the openings G,G. The openings in these screens occupied about $\frac{1}{4}$ of their area. K,K, are floating planks. I,I, are screens made of inch boards; the openings occupied $\frac{1}{3}$ of their area, and consequently the velocity through them was due, theoretically, to a loss of head nine times as great as the head due to the mean velocity of approach. These screens were chiefly relied upon for equalizing the velocity in the different parts of the channel. L,L, are vertical partitions used to direct the flow toward the middle of the channel, and to prevent an eddy from being formed below the pointed pier. M,M, are boards put in after the first trial of the apparatus. N,N, are screens made of $\frac{1}{2}$ inch slats, $\frac{1}{4}$ of an inch apart; the screens were 7.45 feet from the weir.

In constructing the weir many precautions were taken to prevent any change in the level of the crest, either from the swelling of the wood or from insufficient strength. The lower portion of the weir was made of spruce timbers six inches thick, fastened together with bolts running through them from top to bottom; above the spruce timbers, but not touching them, was a hard pine timber, T, 14 inches high, and 7 inches wide; this timber was supported by four vertical hard pine timbers, S,S,S,S, which rested on the masonry, and were bolted only to the top and bottom timbers of the weir, and by two shorter hard pine posts, S',S', resting on the masonry platforms at the ends of the weir.

The spaces between the timbers and the masonry were made water-tight by caulking with oakum; the face of the weir was then sheathed with tongued and grooved boards which fitted into a rabbet in the upper timber. These boards were nailed only to the upper and lower timbers, and care was taken that their ends did not abut against the masonry at the bottom of the channel, nor against the top of the rabbet in the upper timber. All the joints were made water-tight with white lead or elastic cement.

The crest of the weir was made of an iron bar $3\frac{1}{2}$ inches wide, and $\frac{1}{4}$ of an inch thick, firmly screwed to the upper timber. The upper surface of the iron was planed, and the up-stream face was finished with a file

after the bar was secured in place, thus making the up-stream edge sharp.

The form of the upper timber, and the relative positions of the iron bar and the sheathing, is shown in Fig. 5.

The weir was 19 feet long, and 6.55 feet high ; a hook-gauge was fastened to each of the end posts, S', S', which were firmly bolted to the weir. The height of the weir and gauges, as compared one with the other, remained very uniform, the maximum variation in five months being only 0.0002 feet.

X, X, are tin hook-gauge boxes resting on movable shelves which could be clamped at any desired height ; each could be connected at will with either one of three rubber pipes, the first communicating with the orifices O, O, 6 feet from the weir, the second with the pipe P, at the bottom of the channel near the weir, the third with the apparatus (Fig. 5) for comparing the level of the gauges and of the weir. Details of one of the piezometers, O, O, for taking the head 6 feet from the weir, are given in Figs. 3 and 4. They consisted essentially of a small T, flanged at opposite ends with thin steel plates set parallel with each other, and at right angles with the pipe. The T communicated through a $\frac{3}{8}$ inch gas pipe, with a pipe of $1\frac{1}{4}$ inches inside diameter ; the latter extended across the channel, but it was plugged in the middle, and either half was connected with the gauge box on the same side.

The apparatus used for comparing the levels of the hook-gauges and the weir is shown in Fig. 5. It consisted of a piece of hard wood, C, which was hung from the iron crest of the weir, B, by two pieces of metal, one of which is shown at D. The wood served as a support for a steel spring, E, which carried at one end a hook, having a point slightly below the level of the crest when the spring was not in tension, but which could be raised to this level by means of a thumb-screw. The hook was brought to the exact height of the crest by means of a good spirit-level ; then the tin box, F, was put in place without disturbing the hook. The rubber pipes, G, G, led from this box to the hook-gauge boxes. The box F was filled until the surface of the water coincided with the point of the hook E ; then a reading of the hook-gauges at X, X, gave directly the comparison between the zero of the gauge and the crest of the weir.

A peculiarity of the apparatus was that the water would not at all

times settle to the same level in the boxes X, X, and F; a variation of 0.001 feet being quite common. The reason for this was not apparent, as care was taken to run water through the pipes to expel the air and to insure the same temperature of the water at all points before making a comparison.

When such a difference of level was observed, the running of water back and forth through the pipes was continued until it would settle to the same level in all of the boxes. It is possible that this difficulty might have been avoided by the use of larger pipes.

During the experiments of the previous years apparatus constructed on the same principle as that just described has been used, with the exception that the water in the basin above the weir was raised to the level of the hook at the weir, and the tin box and connecting pipes were dispensed with.

The following features were common to all experiments from the beginning:

The gauges were compared with a point in the middle of each foot length of weir; the gauge boxes were blackened inside and lights were placed in such a manner that the reflection took place from the surface of the water close to the point of the hook; to prevent any other reflection all wood-work near the lamps was painted black.

The pipe P, 1½ inches inside diameter, at the bottom of the channel, was perforated on the top with quarter-inch holes, about 8 inches apart; each half was connected with the corresponding gauge box.

Two gates were provided for starting or stopping quickly the flow over the weir. The gate V, on the down-stream side of the weir, shown open in Fig. 1 and closed in Fig. 6, was used to start the flow at beginning of the experiment; when closed it was held in place by fourteen bolts. By means of a system of levers connected with a bar extending the whole length of the gate the bolts could all be withdrawn simultaneously, when the pressure of the water, aided by a counter-weight, forced the gate open very quickly.

The gate U, on the up-stream side, was used to stop the flow at the end of an experiment; it was 19 feet long, and 2.7 feet wide.

The water passing the middle portion of the weir fell upon the platform W, at the mouth of the conduit, and allowed the rest of the water to flow underneath without interruption.

The conduit, from its entrance to the first waste gates, served for a

measuring basin. Only the portion of its area between two water levels 3 feet apart was made use of. The basin was 11 300 feet in length, and nearly nine feet wide, of an approximately rectangular section. The contents of the basin between these levels was 300 272 cubic feet. The height of the water in the basin was measured with gauges at three points, 1 800, 6 000 and 11 300 feet from the weir. The gauges were made of New York leveling rods, with the addition of a metal plate terminating in a point at the bottom of the rod, and a clamp and slow motion screw; the back piece of the rod was screwed to a plank fastened to the masonry at the man-holes where the gauges were operated; two verniers were provided to neutralize inaccuracies in graduation. The length of the portion of the rod most used was compared with a steel standard every day during the experiments.

Nine partitions, each occupying about one-half of the width of the conduit, were provided to prevent oscillations from one end of the measuring basin to the other. After a few experiments some movable parts were added to the partition near the middle of the basin, by which the partition could be made to cover the whole section of the conduit, with the exception of an area of 3 or 4 square feet. By so reducing the area through which the water passed at or near the end of an experiment, the oscillations were made to diminish quite rapidly.

The lower end of the basin was closed with stop-planks, in grooves placed on either side of the conduit. Joints at the end and bottom of the planks were made tight with oakum, and joints between the planks with wicking and white lead.

On account of the long time required for an experiment (one day for each), special care was taken to insure accuracy.

Before commencing an experiment, trials were made to ascertain the openings of the iron regulating gates for a flow corresponding to the desired depth on the weir.

The routine of an experiment was as follows: The water in the measuring basin was drawn to the lower level (generally on the day preceding the experiment) and measurements were taken to its surface, with three gauges, every minute for about an hour. During the same time other assistants were closing and fastening the swinging gates *F*, *F* (Plate III), the gate *V*, on the down-stream side of the weir, and opening the iron regulating gates *C*, *C*, to the proper height. Water was next let into the chamber above the weir through a small opening in one

of the swinging gates until it was nearly full ; then the water rose slowly from leakage until it reached the height which was to be maintained during the experiment. At this moment the signal was given to start, which was accomplished by the sudden opening of the swinging gates and of the gate *V*, at the weir. Two stop-watches were started at the moment the gate was opened, and, in addition, the time was noted with an ordinary watch. It was not thought best to trust the stop-watches for keeping the duration of the experiment, and within a few minutes of the beginning and end they were each compared with three ordinary watches, which were in turn compared with a chronometer before and after each experiment to ascertain their rate. At the end of the experiment the gate *U*, on the up-stream side of the weir, was closed quickly by two men pressing on it, and by the pressure of the water.

The time, at the instant the gate closed, was recorded by four observers, two with stop and two with ordinary watches.

After closing the gate the water in the chamber above the weir rose to the level of the pond ; the iron gates *C*, *C*, were closed as soon as possible.

A very little water leaked past the weir gate after the flow over the weir was stopped ; the amount was measured by observing the rate at which the water above the weir was lowering after the iron gates were closed.

The depths on the weir were measured during the first minute of the experiment with small float-gauges located on either side of the channel, graphical records being taken every five seconds ; during the next two minutes simultaneous measurements were taken with the float and hook-gauges to ascertain the correction to be applied to the former. The float-gauges were then removed, and hook-gauge measurements were taken every minute until the end of the experiment. The depths on the weir were kept very nearly constant by slight movements of the regulating gates *C*, *C* ; few changes were required, however, as a volume equal to that flowing over the weir was drawn into the pond from the storage reservoirs above, thereby keeping its height constant.

Within an hour from the end of each experiment the water in the measuring basin was sufficiently quiet to allow measurements to be taken to its surface ; it was never entirely quiet, however, but always oscillated with a slow movement from one end of the basin to the other.

At the low level the amplitude of the oscillation was small, but at the high level it was several hundredths or, at times, a tenth of a foot during the time that measurements were taken.

At the low level the duration of a full oscillation was 54 minutes, at the high level 31 minutes. Care was taken to select such a number of gauge readings as should cover one or more full oscillations.

The water in the measuring basin was, at either level, below the water in the ground through which the conduit was built. In consequence, there was some filtration through the masonry into the basin. The amount was determined twice at the low level and twice at the high level, by observing the rise in the measuring basin in a given time. The total volume furnished by filtration during an average experiment was about $\frac{1}{500}$ part of the volume measured; this small quantity was so definitely known that no appreciable error in the results was caused by it. To furnish an example of the general features of an experiment, one will now be presented in some detail:

EXPERIMENT NO. 7, NOVEMBER 28TH, 1879.

Observed depth on weir by N. hook-gauge.....	0.9874 ft.
" " " S. " "	0.9853 "
Average observed depth on weir.....	0.9864 "
Correction for velocity of approach.....	+0.0070 "
Mean depth on weir corrected for velocity of approach (H).....	0.9934 "
Length of the weir (L).....	18.996 "

	h.	′	″
Duration of the experiment by each of three watches	1	19	31.8
	1	19	32.3
	1	19	33.2

Average duration of experiment.....1h. 19′ 32.4″ = 4772.4 sec.

		Low Level.		High Level.	
Observed elevation of water in measuring basin above datum plane	Gauge 1..10	A. M.,	141.9664.	1.30 P. M.,	144.9296
	" 2..10	"	141.9655.	1.41 "	144.9275
	" 3 .10	"	141.9662.	1.41 "	144.9244

	Low Level.	High Level.
Mean elevation of water in the basin, deduced from measurements at the three gauges	10 A. M., 141.9660.	1.37 P. M., 144.9274
Corrections for filtration occurring between the time of the observations and the mean time of the experiment, which was 11.07 A. M.	+ 0.0029	—0.0027
Elevations of water in measuring basin reduced to mean time of experiment	141.9689	144.9247

Volume in measuring basin between elevations, 141.9689 and 144.9247	296 056	cu. ft.
Deduct leakage past weir after the flow was stopped	56	" "
Total volume which passed the weir	296.000	" "
Volume passing the weir per second (Q)	62.023	cu. ft.
Value of c in the formula $Q = c\,L\,H^{\frac{3}{2}}$	3.2977	

Table XV., page 69, shows the results of all the experiments made on this weir. For the first calculations of the experiments the formula for velocity of approach, given on page 12, was made use of. The resulting co-efficients are given in column 16, and it will be noticed that they increase with the depths on the weir. This result, which does not seem to accord with theory and experiment, was not unexpected, as it was well known that the arrangements in the channel above the weir (chiefly the short distance between the weir and the screens) were not such as to allow the proper application of our formula for velocity of approach. The experiments were again calculated, using such a co-efficient in the formula for velocity of approach as would make the co-efficients of the weir formula decrease slightly as the depth on the weir increased. The co-efficients thus obtained are given in column 13, and are also plotted on Plate VI. in the vicinity of the curve marked *B*.

It was then found that the formula $Q = 3.291\ L\,H^{\frac{3}{2}} + 0.004\ L$ would represent very nearly the results of these experiments, as can be seen by column 15, which shows the proportional differences between the volumes as actually measured and as calculated by said formula.

TABLE XV.

EXPERIMENTS ON THE FLOW OF WATER OVER A SHARP-CRESTED WEIR, WITHOUT END CONTRACTION, MADE IN THE FARM POND GATE-HOUSE, IN NOVEMBER AND DECEMBER, 1879. DEPTH OF CHANNEL BELOW CREST OF WEIR, 6.55 FEET.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Number of the Experiment.	Date of the Experiment.	Temperature of the Water.	Observed Depth on the Weir.	Mean Velocity of Approach.	Theoretical Head due to Mean Velocity of Approach (h).	Correction for Velocity of Approach by the Formula $\epsilon = 2.4\,h$.	Depth on the Weir corrected for Velocity of Approach (H).	Length of Weir (L).	Total Volume of Water which passed the Weir during the Experiment.	Duration of the Experiment.	Volume of Water passing the Weir per Second (Q).	Value of c in the Formula $Q = c\,L\,H^{\frac{3}{2}}$.	Volume of Water Passing the Weir calculated by the Formula $Q = 3.291\,L\,H^{\frac{3}{2}} + 0.004\,L$.	Proportional Difference between the Volume as Measured and as Calculated, using the Calculated Volume as the Basis.	Value of c when the Correction for Velocity of Approach is Calculated from the Formula given on page 12.
	1879.	Degrees Farenh't.	Feet.	Feet per Sec.	Feet.	Feet.	Feet.	Feet.	Cubic Feet.	Seconds	Cubic Feet.		Cubic Feet per Second.		
1	Dec. 1st...	34	1.6038	0.840	0.01097	0.0263	1.6301	18.996	285 397	2193.4	130.117	3.2912	130.186	— 0.0005	3.3269
2	Nov. 26th..	37	1.4546	0.737	0.00844	0.0203	1.4749	"	299 431	2671.9	112.066	3.2936	112 054	+ 0.0001	3.3233
3	" 25th..	37	1.2981	0 632	0 00621	0.0149	1.3130	"	288 087	3058.5	94 192	3.2958	94.132	+ 0.0006	3.3196
*4	" 24th..	37	1.1704	0.549	0.00469	0.0113	1.1817	"	298 914	3711.9	80.529	*3.3001	80.382	*+ 0.0018	*3.3199
5	" 29th..	39	1.1456	0.532	0.00440	0.0106	1.1562	"	303 447	3901.2	77.783	3 2936	77.797	— 0.0002	3.3125
6	" 20th..	43	0.9873	0.433	0.00291	0.0070	0.9943	"	296 375	4775.5	62.061	3.2952	62.058	0.0000	3.3097
7	" 28th..	38	0.9864	0.433	0.00291	0.0070	0.9934	"	296 000	4772.4	62.023	3.2977	61.974	+ 0.0008	3.3122
8	" 17th..	45	0.8191	0.334	0.00173	0.0042	0.8233	"	300 769	6432.2	46.760	3.2952	46.777	— 0.0004	3.3054
9	" 19th..	44	0.6460	0.239	0.00089	0.0021	0.6481	"	297 859	9112 9	32.685	3.2978	32.694	— 0.0003	3.3040
10	" 21st..	41	0.4685	0.151	0.00035	0.0008	0.4693	"	299 676	14851.9	20 178	3.3040	20.175	+ 0.0001	3.3071

* In Experiment No. 4 a leak occurred through the stop-planks at the lower end of the measuring basin. The quantity of water lost was roughly estimated from data furnished by the assistant stationed at this point, but the experiment was considered defective, and another (No. 5) was made with nearly the same depth on the weir.

The maximum difference in this column is in Experiment 4 (the defective experiment), and is $\frac{1}{10}\frac{8}{0}$ of one per cent. The other differences in no case exceed $\frac{1}{10}\frac{8}{0}$ of one per cent., and average about $\frac{3}{10}\frac{3}{0}$ of one per cent., which shows that these experiments agree very closely among themselves; but by comparing them with the experiments made on the weir 5 feet long (see Table XIV., page 56), it will be seen that there is a very noticeable difference in the results, the co-efficients with the large weir being more than one per cent. smaller than with the other. This discovery led to an investigation to ascertain, if possible, the cause of the difference, but none was found.

A few of the more important points covered by this investigation, also some of the methods employed in carrying on the experiments, will now be considered.

Capacity of the Measuring Basin.

The conduit has an inclination of one foot per mile; the measuring basin was a portion of the conduit included between two water surfaces. The contents of the basin within the theoretical cross-section of the conduit were susceptible of calculation by the prismoidal formula; but to avoid any doubt about its applicability, the contents of the various geometrical figures into which the basin could be divided were also calculated, and the same final result was obtained. The conduit was carefully built to the proper form, but the difference between its theoretical and actual section was ascertained in the following manner:

One year and a half before the experiments, diagrams of the conduit were taken, graphically, every 25 feet in its length. These diagrams showed to the full size the difference between the theoretical and actual sections of the conduit, and from them an increase of $\frac{1}{2}$ of one per cent. in the capacity of the basin over its theoretical value was found.

At the time of the experiments, an additional series of measurements were made to detect further change in the shape of the conduit. A slight increase in width was thus found, which added $\frac{1}{4}$ of one per cent. to the capacity previously determined. The necessary additions were made for portions of the gate-house and waste-weir included in the basin, and the proper deductions were made for various timbers.

To check the capacity of the basin, the measured widths of the conduit were used without reference to the diagrams, by assuming that the

ratio between the theoretical and actual capacity of the basin was the same as between the theoretical and actual widths at the springing line. The result thus obtained was practically identical with the other. The length of the basin was a well-checked distance.

The height that the water rose was taken by three observers with independent apparatus.

The last thing considered was the possibility of absorption of water by the brick-work. The basin was below the level of the water in the ground for from 90 to 95 per cent. of its length, and water percolating into the conduit trickled down its sides; for the remainder of the length it was thought that bricks would remain nearly saturated by absorbing water from the damp air of the conduit, and by being soaked each time the basin was filled. In some cases the bricks were soaked by water standing at the high level in the basin but a few hours before an experiment, while at other times the water was drawn down a day or two in advance; yet it occasioned no perceptible difference in the results. To obtain some idea of the error which might be due to this absorption, the following experiment was made:

A brick, which had previously been thoroughly dried, when immersed in water, absorbed 12 per cent. of its volume. After being taken out of the water it was left standing upon end in a damp place for 24 hours, when it was again immersed, and it absorbed 1½ per cent. of water. Using the percentage last obtained as a basis, it has been found that the absorption of water by the brick-work was not a sufficiently important factor to affect the results appreciably.

The study of the capacity of the basin has been thus described in detail because it seemed the most probable place to look for an important error affecting all of the experiments.

Velocity of Approach.

On account of the peculiar form of the channel it was uncertain whether or not the water surface would be level in different parts of its width, and for this reason it was thought advisable to take the head on the weir from four points, each being nearly in the middle of a fourth part of this width. The piezometers (described on page 63) were designed to measure the head at these points without being affected by the velocity of the water passing the orifices, and it was consequently necessary that the steel plates should be parallel with the current.

To accomplish this result a hook-gauge was put in communication with a single piezometer, and the head on the weir was taken alternately at each orifice while the other was closed. If the head taken at one orifice exceeded that taken at the other, it was evident that the plate containing the former faced the current somewhat, as the difference in heads at the two orifices was the sum of the increased head and of the suction due to the velocity of the water. From data furnished by several trials of this kind, each piezometer was adjusted so that its two orifices furnished approximately the same head. The adjustment was rather a tedious operation, and great accuracy was not attempted, as the direction of the currents was liable to vary with the depths on the weir, and because the construction of the piezometers was such that suction at one orifice was compensated to a large extent by the increased pressure at the other.

The following table shows a series of measurements made after the final adjustment of the piezometers:

	Depths on the Weir.
North gauge—All orifices open	1.3046 feet.
" " North piezometer, north orifice	1.3024 feet.
" " " " south "	1.3021 "
" " South " north "	1.3100 "
" " " " south "	1.3077 "
Average of heads taken separately	1.3055 feet.
South gauge—All orifices open	1.3041 feet.
" " South piezometer, south orifice	1.3054 feet.
" " " " north "	1.3007 "
" " North " south "	1.3089 "
" " " " north "	1.3074 "
Average of heads taken separately	1.3056 feet.

Mean velocity of approach, 0.63 feet per second.
Theoretical head due to mean velocity, 0.0062 feet.

On account of the short distance between the screens and the weir, it was known that the shape of the angle of pressure near the weir would be modified. To ascertain the excess of head within the angle, simulta-

neous measurements were taken of the head within it, and at the orifices 6 feet from the weir.

Table XVI. shows the results of these measurements.

TABLE XVI.

MEASUREMENTS SHOWING THE EXCESS OF HEAD TAKEN IN THE ANGLE NEAR THE WEIR, AS COMPARED WITH THE HEAD TAKEN 6 FEET FROM THE WEIR.

1 Number of the Experiment.	2 Date of the Experiment.	3 Head on the Weir taken at the Orifices 6 feet from the Weir.	4 Head on the Weir taken by Perforated Pipe in the Angle near the Weir.	5 Excess of Head taken in the Angle.	6 Mean Velocity of Approach.	7 Theoretical Head due to the Mean Velocity of Approach. (h)	8 Co-efficient in Formula $E = c\,h$.	9 REMARKS.
	1879.	Feet.	Feet.	Feet.	Ft. per sec.	Feet.		
1	Nov. 22..	0.9918	0.9996	0.0078	0.436	0.00296	2.64	Piezometers 6 feet from weir approximately adjusted.
2	" ..	0.8248	0.8290	0.0042	0.338	0.00178	2.36	
3	" ..	1.1606	1.1720	0.0114	0.540	0.00453	2.52	Piezometers finally adjusted.
	1880.							
4	April 9..	1.6093	1.6367	0.0274	0.842	0.01102	2.49	

The results as given in column 8 show that the excess of head within the angle is practically constant, and equals 2½ times the theoretical head due to the mean velocity of approach. This large excess may be due in part to the fact that the head taken outside of the angle could not be taken sufficiently far from the weir to be above the point where, under normal conditions, the surface curvature would begin.

The difference between the heads in the two halves of the channel was from two to three times as great when taken within the angle, as when taken at the piezometers 6 feet from the weir.

The distribution of velocities in different parts of the channel was measured with a current meter in a vertical plane 5 feet from the weir. Mean velocities at different heights above the bottom of the channel were obtained by holding the current meter at a constant level, and moving it slowly at a uniform rate from one side of the channel to the other.

Mean velocities in vertical lines at different distances from the sides of the channel were obtained in a similar manner, and are also recorded in Table XVII.

TABLE XVII.

VELOCITIES IN CHANNEL OF APPROACH 5 FEET FROM WEIR.

Height above Bottom of Channel.	Mean Velocity of Approach at the Height given in the Previous Column.	Distance from South side of Channel.	Mean Velocity of Approach at the Distance given in the Previous Column.	Distance from North side of Channel.	Mean Velocity of Approach at the Distance given in the Previous Column.
Feet.	Feet per sec.	Feet.	Feet per sec.	Feet.	Feet per sec.
7.7	0.713	0.5	0.608	0.5	0.666
6.7	0.720	1.5	0.563	1.5	0.669
5.7	0.632	2.5	0.596	2.5	0.593
4.7	0.643	3.5	0.579	3.5	0.529
3.7	0.672	4.5	0.566	4.5	0.566
2.7	0.552	5.5	0.580	5.5	0.577
1.7	0.643	6.5	0.601	6.5	0.614
0.7	0.394	7.5	0.714	7.5	0.708
....	.....	8.5	0.806	8.5	0.789
		Middle of Channel.			
....	.....	9.5	0.928	....	.....

Mean depth on the weir ..... 1.304 feet.
Mean velocity of approach by weir measurement. 0.63 feet per sec.

The results as given in the table are thought to be quite satisfactory considering the short distance which was available for regulating the velocities. It will be observed that the maximum velocity was in the middle of the channel directly in front of the stone pier.

Bubbles of air, caused by the rush of water immediately below the gates, came through the screens, and with the larger velocities some of them passed the piezometers before rising to the surface; these bubbles may have had a slight effect upon the curvature of the sheet, and consequently upon the correction of the depth on the weir for velocity of approach.

A swelling of the sheet occurred at the ends of the weir; adjoining it was a depression of nearly equal extent; the effect noticed is caused by water which enters the angle of pressure where the velocity of approach is greatest, and finds its way to the surface at the ends of the weir in the manner described on page 43. It is not known to what extent this affects the discharge.

From the various statements made concerning the abnormal conditions existing in the channel of approach, it will be seen that they were sufficient to render doubtful the applicability of any known formula for velocity of approach. There were several indications that this correction should be larger than it would be under normal conditions, but nothing indicated the extent of this increase, consequently the correction used was, as we have already stated on page 68, the one which seemed best suited to the experiments themselves.

This correction was found to be furnished by the formula $c = 2.4\ h$, and is larger than that obtained from the regular formula given on page 12 by very nearly the theoretical head due to the mean velocity of approach. Although this is a large difference in the formula for velocity of approach, yet its effect upon the co-efficient in the weir formula is but one per cent. with the largest depth on the weir and $\frac{1}{10}$ of one per cent. with the smallest depth.

It seems improbable that the error due to velocity of approach exceeds the percentages here given; such an error is far from being sufficient to explain the difference between the formula deduced from the experiments on the weir 5 feet long and on the weir 19 feet long, and some other reason must exist for the discrepancy; but a careful study of all the quantities entering into the computations, and of all known circumstances affecting the discharge, failed to reveal in either case a cause for any important inaccuracy in the results, yet it is evident from the difference in the formulæ, that there was, in one or both series, some nearly constant error, or some important cause affecting the flow over weirs, which remains unknown. Notwithstanding the differences found between the two series of experiments, they agree closely enough to furnish a basis for what would be termed an accurate hydraulic formula, but in view of the very accurate experiments made by Mr. Francis, we consider that the experiments with depths on the weir smaller than those used by him are the ones having the chief practical value.

The formula already given for the small weir $Q = 3.33\,L\,H + 0.0065\,L$ could be used for these smaller depths, but as it would be inconvenient to use two different formulæ (one for small depths and one for large depths on the weir), especially in view of the fact that they would give different results for depths which are common to both series of experiments, a general formula was sought which would cover all depths within experimental limits.

In establishing this formula we made use of the data furnished by the experiments of Mr. Francis, using in their computation the additional knowledge in regard to the effect of velocity of approach furnished by our investigations of that subject.

The experiments* of Mr. Francis, which we propose to review, consist of many measurements of the discharge, with each of six diverse forms of apparatus, but so far as velocity of approach is concerned, they may be divided into two classes, viz.: those made upon weirs with and without end contractions.

The use of our formulæ for velocity of approach being limited by certain conditions, as given on page 51, it becomes necessary to examine the conditions which existed during these experiments, in order to test the applicability of these formulæ to them.

In the case of the weir with end contractions, it is found that the channel of approach is not a "straight channel of uniform section," but varies from it chiefly by widening abruptly from about 12 feet to 13.96 feet, a little more than 6 feet from the weir. This widening, by permitting the formation of eddies, may have modified the excess of head in the angle of pressure, and otherwise affected the application of our formula.

The bottom of the channel inclined gradually upward toward the weir, but this fact seems to have less importance.

It is not certain that the conditions relating to the place and manner of taking the head were fully complied with. Since this examination of the conditions existing during these experiments shows that we would not be justified in applying our own formulæ, it is fortunate that a datum for making the correction for velocity of approach may be supplied from certain of the experiments now under consideration.

The datum referred to is obtained by comparing certain series of ex-

* Lowell Hydraulic Experiments, page 103, *et seq.*

periments* which correspond with each other in every important particular except in the depth of the channel of approach.

In determining the correction for velocity of approach from the comparison of two experiments in which the depth of the channel of approach is the only important variable, it may be assumed without much error that the co-efficients in the formula for discharge ($Q = cLH^{\frac{3}{2}}$) and in the formula for velocity of approach ($C = ch$) are constant.

Calculations made upon this basis, by comparing Series 3 with Series 5 (see Table XVIII., page 79), in which the mean depth on the weir is about 1.03 feet, gave $1.47h$ as the height to be used for the correction for velocity of approach.

A similar comparison between Series 8 and 9, with a mean depth on the weir of 0.82 feet, gave 1.33 h, and between Series 11 and 12, with a mean depth on the weir of 0.64 feet, 0.89 h.

The variation in the results found does not seem to be the result of any law, but rather of the inherent difficulty of measuring within a small percentage the minute quantities which represent the effect of velocity of approach.

For this reason an average of the results has been taken for our use, but in making this average each of the co-efficients was allowed weight in proportion to the size of the small quantities referred to.

By this method the average correction was found to be 1.35 h, which will be used in computing the experiments upon the weirs with end contractions.

In the case of the weir without end contractions the sides of the channel were straight, and its section was uniform, except for the change due to the inclination of the bottom, which has little importance.

Pipes ending at the bottom of the channel, 6 feet from the weir, were used for taking the head upon the weir in nearly every experiment, as may be seen by comparing the time of the experiments given in Table XIII. with that given in Tables XVII. or XVIII.†

By this method the head was probably taken within the angle of pressure, though not where the pressure was greatest. As a result of this examination of the conditions existing at this form of weir, it is found that our formula for velocity of approach might be applied if we

* These experiments are given in a condensed form in Table XVIII., page 79; their details being given in Lowell Hydraulic Experiments, Table XIII., pp. 122–125.

† The tables referred to may be found in Lowell Hydraulic Experiments, pp. 122–140–142.

could determine a correction for the difference between the heads taken and the heads existing outside of the angle of pressure. Some experiments are recorded in Mr. Francis' work by which this can be done.

From two series of experiments given in Table XVIII.,* with depths on the weir of 0.81 and 0.98 feet, it was found that the head taken through the pipe ending 6 feet from the weir was greater than that taken 12 feet from the weir by $0.40h$ and $0.39h$ respectively. Other comparisons (recorded on page 141 of Lowell Experiments) were made between the heads taken by the pipe 6 feet from the weir and an orifice in the side of the channel near the surface, the same distance from the weir, and they show no practical difference between the heads taken at the two places.

The two methods of comparison seemed about equally good, and the average of the results found, equal to $0.2\ h$, was used to modify our formula.

From an examination of the co-efficients of our formula, as given in Table I., page 12, it will be seen that the co-efficient 1.45 is the most suitable one. Deducting $0.2\ h$ on account of the excess of pressure at the place where the head was measured, there remains $1.25\ h$ as a correction for velocity of approach, to be applied to all of the experiments on this weir without end contractions.

The correction for the effect of the end contraction was made, according to Mr. Francis' rule, by deducting from the measured length of the weir, one-tenth of the depth upon it for each end contraction.

In a similar manner, in two series of experiments in which end contractions were suppressed, but in which the sheet was allowed to expand immediately after passing the weir, a correction was made for this effect by adding to the measured length of the weir one-fiftieth (more exactly, 0.021) of the depth on the weir for each end expansion. The amount of this correction was found by comparing Series 6, in which end expansion existed, with Series 7, in which it was suppressed.

Table XVIII., on the following page, gives the results of the experiments as calculated with the new corrections which have just been described. The results are given in series, each series being the mean of a group of experiments made under nearly identical circumstances.

Details of each experiment may be found in "Lowell Hydraulic Experiments," Table XIII., pages 122–125.

* Lowell Hydraulic Experiments, p. 142.

TABLE XVIII.

EXPERIMENTS ON THE FLOW OF WATER OVER WEIRS MADE BY MR. J. B. FRANCIS, IN 1852, AS RECALCULATED WITH A NEW CORRECTION FOR VELOCITY OF APPROACH.

1 Number of the Series.	2 Numbers of the Experiments.	3 Numbers referring to the Conditions of the Experiments as given in the Foot Notes.	4 Mean Depth upon the Weir by Observation.	5 Theoretical Head due to the Mean Velocity of Approach. (h)	6 Depth upon the Weir corrected for Velocity of Approach. (H)	7 Length of the Weir corrected for end Contractions or Expansions. (L)	8 Value of c in the Formula $Q = cLH^{\frac{3}{2}}$ from Experiment.	9 Value of c in the Formula $Q = cLH^{\frac{3}{2}}$ calculated from the Formula $Q = 3.313\ LH^{\frac{3}{2}} + 0.006\ L$.	10 Proportional Differences of Co-efficients in Columns 8 and 9, using those in Column 8 as the Basis.
			Feet.	Feet.	Feet.	Feet.			
1	1– 4	1	1.55079	0.00950	1.56362	9.6843	3.3062	3.3161	+0 0030
2	5–10	1	1.24757	0.00552	1.25502	9.7460	3.3253	3.3173	—0.0024
3	11–33	1	0.99732	0.00306	1.00145	9.7967	3.3165	3.3190	+0.0008
4	34–35	5	1.01825	0.00197	1.02091	7.5886	3.3565	3.3188	—0 0112
5	36–43	4	1.05033	0.01475	1.07024	9.7830	3.3235	3.3184	—0.0015
6	44–50	3	0.97900	0.00454	0.98467	10.0364	3.3202	3.3191	—0.0003
7	51–55	2	1.00026	0.00477	1.00622	9.9950	3.3200	3.3189	—0.0003
8	56–61	1	0.79899	0.00168	0.80126	9.8367	3.3207	3.3214	+0.0002
9	62–66	4	0.82624	0.00831	0.83746	9.8295	3.3198	3.3208	+0.0003
10	67–71	3	0.79517	0.00258	0.79840	10.0286	3.3234	3.3214	—0.0006
11	72–78	1	0.62355	0.00086	0.62471	9.8721	3.3249	3.3252	+0.0001
12	79–84	4	0.64928	0.00451	0.65537	9.8659	3.3124	3.3243	+0.0036
13	85–88	6	0.68004	0.00310	0.68422	7.7233	3.3281	3.3236	—0.0014

Conditions of the experiments referred to by the numbers in column 3 :

1 indicates the weir with two end contractions and deep channel.
2 " " " without " " " or expansions and with deep channel.
3 " " " " " " but with end expansions and deep channel.
4 " " " with two " " and shallow channel.
5 " " " " four " " and deep channel.
6 " " " " " " and shallow channel.

Description of Table XVIII.

The number of the experiments given in column 2 are the same as those given in column 1 of the original table.

Columns 4 and 5 were obtained by a direct average of the quantities given in columns 7 and 9 of the original table.

Column 6 was obtained by applying the new corrections for the velocity of approach to the depths given in column 4.

The lengths of weir given in column 7 were obtained by correcting the lengths given in column 11 of the original table, in the manner just stated.

The co-efficients given in column 8 correspond to those given in column 16 of the original table, and were deduced from them by making the necessary modifications for the slight changes in depth on, and length of, the weirs occasioned by the new corrections for velocity of approach and end expansions. This method of recalculating the experiments by series was adopted because it gave all the results needed with little labor, and with almost exactly the same degree of accuracy as if each experiment had been taken separately.

Column 9. The formula $Q = 3.313\, L\, H^{\frac{3}{2}} + 0.006\, L$ was selected to represent the results of these experiments, and from it the values of c in the formula $Q = c\, L\, H^{\frac{3}{2}}$ have been calculated for each series of experiments.

Column 10, which corresponds to column 20 of the original table, gives the proportional differences resulting from the comparison of columns 8 and 9, or, in other words, the comparison between the results of the experiments and of the formula. The average of these proportional differences, signs being disregarded, is in this table 0.0020; in the original table 0.0029. The comparison will be more fairly made, however, if Series 5, 9, 12 and 6 are omitted, since we have used them to obtain the corrections for velocity of approach and end expansions, and, in consequence, they may agree more closely than they otherwise would. With these omissions the average of the differences is, in this table 0.0022; and in the original table 0.0027.

These comparisons indicate that the use of the larger correction for velocity of approach in connection with the new form of weir formula, though based upon experiments of our own, is also in accordance with the experiments of Mr. Francis.

The results given in the table are also shown on Plate VI.

Curve "C" represents the formula adopted, and the black circles in its vicinity show the co-efficients deduced from the experiments. The figures attached to the circles indicate the conditions of the experiments, and are the same as those given in column 3 of the table.

Explanation of Plate VI.

The experiments made upon the weir 19 feet long are shown by a half-filled circle, and the formula $Q = 3.291\ L\ H^{\frac{3}{2}} + 0.004\ L$, representing them, is shown by the curve marked *B*.

The experiments made upon the weir 5 feet long when measured in the "large basin" are shown by an open circle; when measured in the "small basin" they are shown by crosses. The formula

$$Q = 3.33\ L\ H^{\frac{3}{2}} + 0.0065\ L,$$

representing these experiments, is shown by the curve marked *A*.

The experiments made by J. B. Francis upon various forms of weirs are shown by filled circles, and the figures accompanying them indicate the condition of the weirs used, as per the following schedule, corresponding to the foot-note on Table XVIII.

Weir with 2 end contractions and deep channel	1
" without end contractions or expansion with deep channel	2
" " " " with end expansion and deep channel	3
" with 2 " " and shallow channel	4
" " 4 " " " deep "	5
" " 4 " " " shallow "	6

The formula $Q = 3.313\ L\ H^{\frac{3}{2}} + 0.006\ L$, representing these experiments, is shown by the curve marked *C*.

The formula $Q = 3.31\ L\ H^{\frac{3}{2}} + 0.007\ L$, finally adopted, is shown by the broken line marked *D*.

THE FINAL FORMULA.

Thus far we have presented three different formulæ for the discharge of water over weirs, as follows :

Formula deduced from experiments upon weir 5 feet long,

$$Q = 3.33\ L\ H^{\frac{3}{2}} + 0.0065\ L.$$

Formula deduced from experiments upon weir 19 feet long,

$$Q = 3.291\ L\ H^{\frac{3}{2}} + 0.004\ L.$$

Formula deduced from Mr. Francis' experiments,

$$Q = 3.313\ L\ H^{\frac{3}{2}} + 0.006\ L.$$

From an examination of all these results, the formula

$$Q = 3.31\ L\ H^{\frac{3}{2}} + 0.007\ L$$

was finally adopted to represent the discharge over a weir unaffected by end contractions or velocity of approach ; if these conditions exist, the corrections for them must be made separately.

This formula is represented by the curve "*D*" on Plate VI., where its comparison with the different series of experiments may be seen ; it agrees closely with the results of Mr. Francis' experiments; is about $\frac{2}{3}$ of one per cent. larger than the formula for the large weir, and is about $\frac{6}{10}$ of one per cent. smaller than the formula for the small weir when the depth is 0.8 feet. For the small depths it is based almost entirely upon the experiments made with the small weir, 5 feet long. In view of the rapid increase of the co-efficients with the smaller depths, it was thought to be very desirable to check our results by the experiments of others, and some of those on record were examined for this purpose.

Mr. Francis has recorded a series of experiments* in which the discharge of a constant volume of water over weirs of different lengths was compared.

From this series five cases are selected in which the discharge was least influenced by end contractions; in each case, assuming the co-efficient calculated from our final formula to be correct for the larger depth, the co-efficient for the smaller depth was calculated from the experiment.

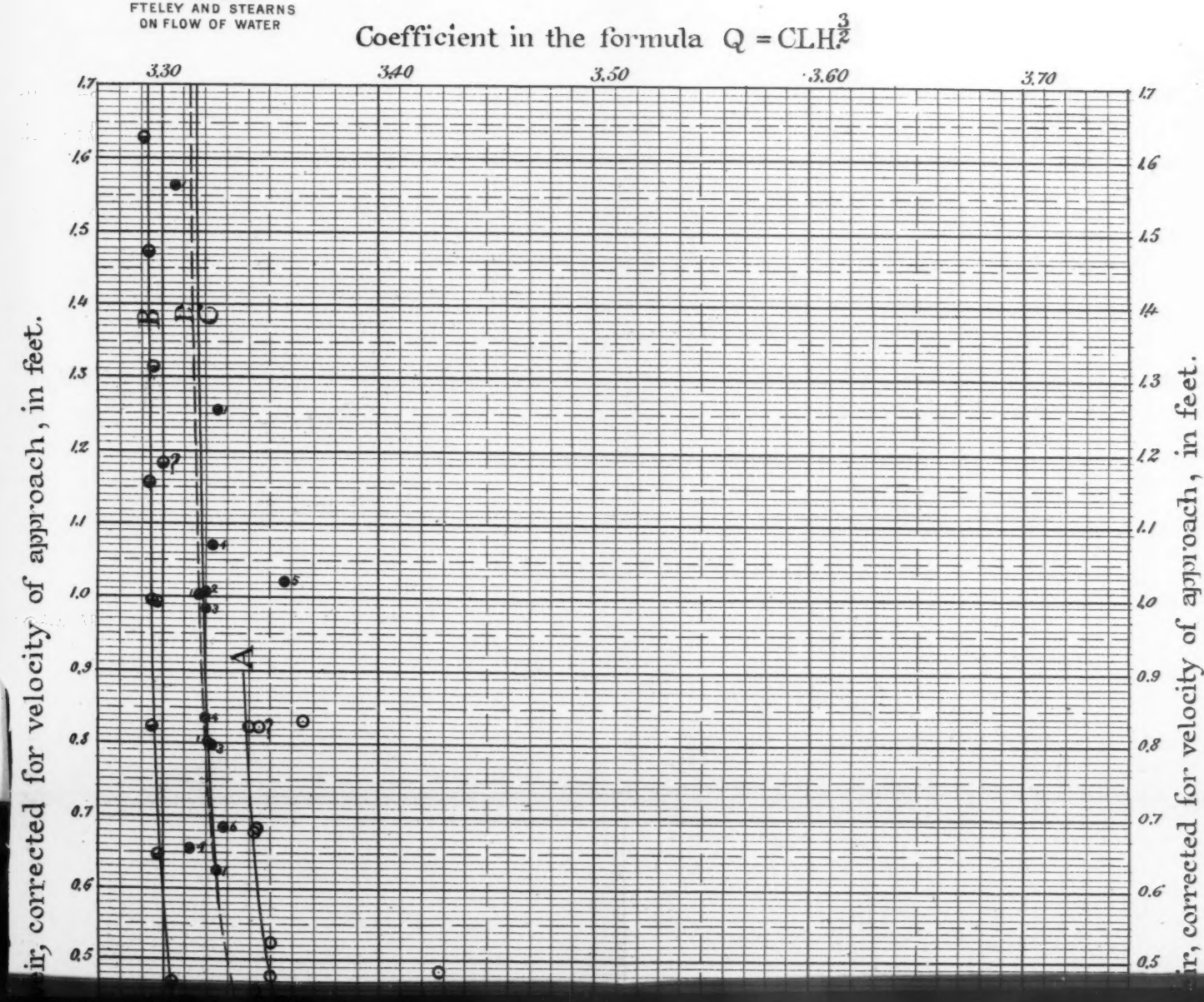
The correction for end contractions was made by Mr. Francis' rule. The results are given in the following table :

* Lowell Hydraulic Experiments, page 88.

PLATE VI.

DIAGRAM SHOWING EXPERIMENTS ON THE DISCHARGE OF WATER OVER SHARP CRESTED WEIRS.

TRANS. AM. SOC. CIV. ENGRS
VOL. XII. No. CCLIII.
FTELEY AND STEARNS
ON FLOW OF WATER



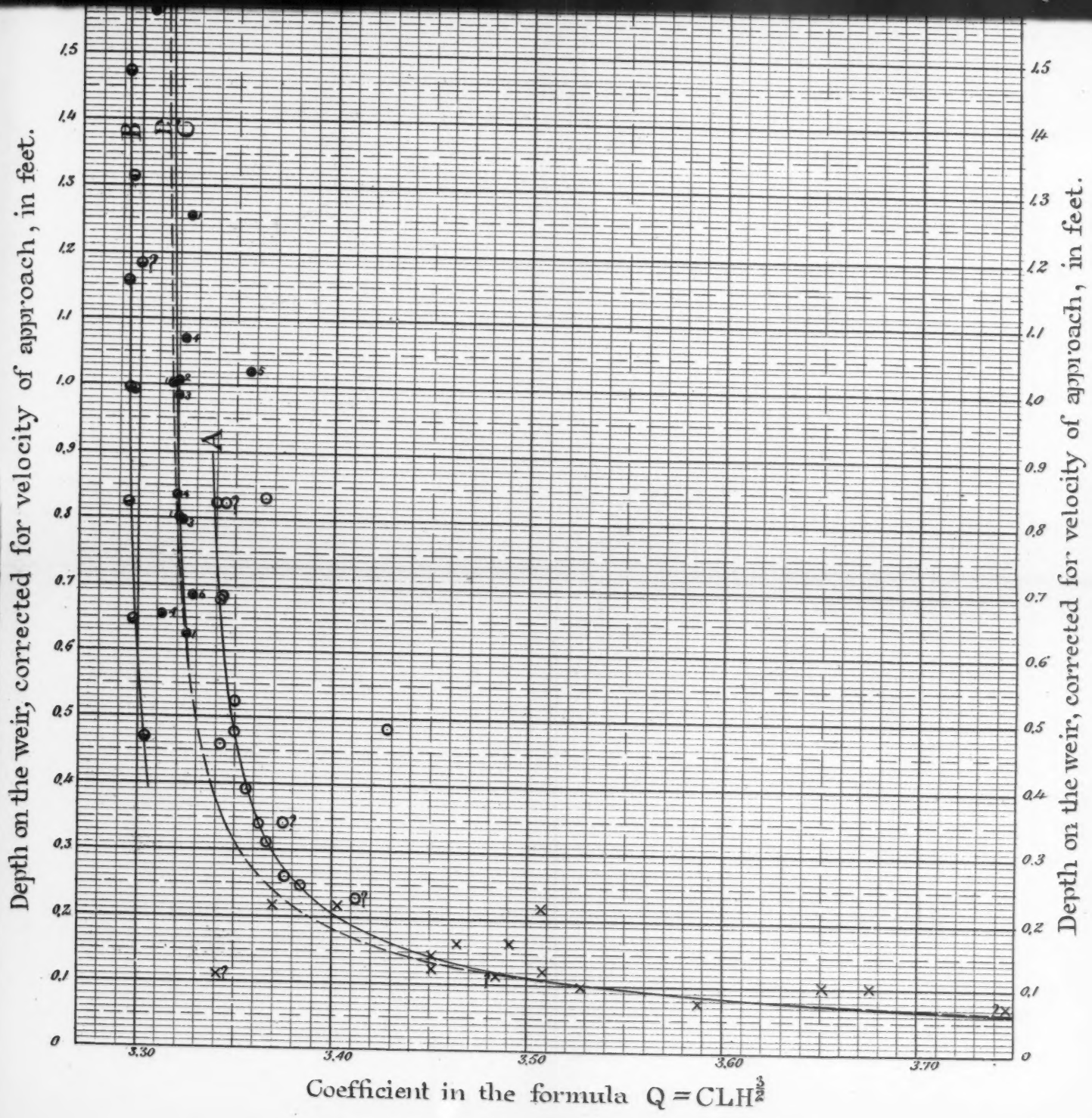

Depth on the weir, corrected for velocity of approach, in feet.
Depth on the weir, corrected for velocity of approach, in feet.
Coefficient in the formula $Q = CLH^{\frac{3}{2}}$
3.30
3.40
3.50
3.60
3.70
A
B
C
D

TABLE XIX.

1 Mean Depth on the Shorter Weir.	2 Value of c in the formula $Q = cLH^{\frac{3}{2}}$ Calculated from the formula $Q = 3.31\ LH^{\frac{3}{2}} + 0.007\ L$.	3 Mean Depth on the Longer Weir.	4 Value of c in the formula $Q = cLH^{\frac{3}{2}}$ Calculated from the formula $Q = 3.31\ LH^{\frac{3}{2}} + 0.007\ L$.	5 Co-efficient deduced from the Experiment made by Mr. Francis.
Feet.		Feet.		
0.6554	3.3232	0.3584	3.3426	3.3463
0.4715	3.3316	0.2917	3.3544	3.3588
0.4681	3.3319	0.2552	3.3643	3.3804
0.3745	3.3405	0.2339	3.3719	3.3693
0.4936	3.3302	0.2282	3.3744	3.3797

A comparison of columns 2 and 4 shows the increase of co-efficients with smaller depths as given by our formula; a comparison of columns 2 and 5 the increase as given by the experiments of Mr. Francis.

Although the results agree quite nearly, it may be noticed that the experiments show a somewhat more rapid increase than the formula.

An examination of some of the older experiments on record indicated that a comparison with them would have much less value, as they were generally made upon short weirs having end contractions and with apparatus less perfect than those used at the present day. We did not make any extended comparisons with these experiments, in view of the fact that they have been examined by Gen. T. G. Ellis, M. Am. Soc. C. E., who writes concerning them as follows*: "A comparison was made between Francis' formula and the results of experiments by Poncelet and Lesbros and other observers, taken with smaller heads to ascertain their differences." "The discharge was in all cases greater with the lower heads, commencing at 0.25 feet in depth on the weir and increasing to 6 per cent, more than the amount given by Francis' formula when the depth on the weir was 0.09 feet."

This agrees quite well with our formula, which gives 7 per cent. more than Mr. Francis' formula at the latter depth on the weir.

*Description and Results of Hydraulic Experiments with large Apertures, at Holyoke, Mass. Transactions of the Society, Feb., 1876, page 47.

The formula $Q = 3.31\ L\ H^{\frac{3}{2}} + 0\ 007\ L$, which we have finally adopted, is based upon experiments in which the depths on the weir ranged from 0.07 to 1.63 feet.

The former depth we would consider the smallest to which the formula would apply; but in view of the marked degree of constancy in the co-efficients with the larger depths, there seems to be no reason to doubt that it would apply to as large depths on the weir as could be properly corrected for the effect of velocity of approach.

There are reasons for thinking that the increase in the co-efficients with the smaller depths may be largely due to the modification of the bottom contraction of the sheet, caused by the cohesion of the water and by its adhesion to the up-stream face and edge of the weir. If this is the case, the co-efficients for the smaller depths may vary with the temperature of the water, which affects its cohesion, and the adhesion may vary with the material of which the crest of the weir is made, although some experiments* made by Hagen indicate that it would not.

The experiments referred to showed that water adhered to a dead polished brass surface and to surfaces of box wood, slate and glass with equal force.

For the reasons given above, as well as for the more obvious one, that errors due to the influence of wind and to the measurement of the depth on the weir are proportionately greater with the smaller depths, it is desirable to maintain a larger depth by shortening the weir.

The variations of the results of our formula $Q = 3.31\ L\ H^{\frac{3}{2}} + 0.007\ L$, from the results of Mr. Francis, may be seen by reference to Table XX.

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* The Mechanics of Engineering, by Julius Weisbach, Vol. I. Translated by Eckley B. Coxe, page 770.

TABLE XX.

PERCENTAGE VARIATION OF THE FORMULA $Q = 3.31\ L\ H^{\frac{3}{2}} + 0.007\ L$, FROM THE FORMULA $Q = 3.33\ L\ H^{\frac{3}{2}}$ WHEN THERE IS NO VELOCITY OF APPROACH.

Depth on the Weir.	Percentage Variation from Mr. Francis' Formula.
Feet.	Per Cent.
Infinite.	0.6 less.
1.634	0.5 "
0.497	0.0
0.258	1.0 more.
0.187	2.0 "
0.150	3.0 "
0.128	4.0 "
0.112	5.0 "
0.100	6.0 "
0.091	7.0 "
0.084	8.0 "

This table shows that above six inches, which is the smallest depth to which Mr. Francis considers his formula applicable, the variation does not exceed 0.6 of one per cent. when there is no velocity of approach. If, however, the two formulæ are compared in connection with their respective corrections for the effect of velocity of approach, which always occurs in practice, it is found that with small velocities of approach their results are nearly identical. With great velocities the formula $Q = 3.31\ L\ H^{\frac{3}{2}} + 0.007\ L$ gives the larger results.

EXPERIMENTS UPON THE FLOW OF WATER OVER WEIRS WITH WIDE CRESTS.

In March, 1877, while one of the weirs already described was in the Sudbury River Conduit, some observations were taken of the flow of water over crests 4 inches and 6 inches in width, for the original purpose of testing the accuracy of measurements previously made in connection with the water supply of Boston, over flash-boards and stop-planks of the same thickness.

These observations were subsequently extended to a series of experiments with crests from 2 inches to 10 inches in width.

The apparatus used is, with a few exceptions, that shown upon Plate I., and already described upon pages 52 to 53.

The principal differences were in the length of the channel, which was but 6 feet in this case, and the crest of the weir, instead of steel, was of hard pine, with the upper edge as sharp as practicable.

Fig. 5 shows a section of the weir upon a large scale. The dotted lines show the crest widened to six inches.

The piece used to widen the crest could be easily attached or detached from the weir. When slid into place, it was drawn into close contact with the weir by fastenings at five points in its length. The outer edge was supported upon wedges by means of which its height could be adjusted until a spirit level showed the crest to be horizontal.

Other widths of crest were obtained in a similar manner.

The experiments were made by running a constant volume of water over the sharp-crested weir, and then over the same weir after the crest had been widened. The difference in the depth on the weir caused by the widening was noted. The depth on the sharp-crested weir was generally measured before and after an observatien on a wide crest, but sometimes the depths on two of the wide crests were taken in succession.

The widths of the crests were measured at five points in their length, about twice in a week, to detect changes due to the swelling of the wood.

The result sought was to find a law by which the depth on a wide crest could be corrected so as to allow the use of the ordinary formulæ for sharp-crested weirs. Each experiment furnished a correction such as was desired by merely subtracting the depth on the wide crest from the depth on the sharp crest.

The results of the experiments were plotted, using the corrections so obtained for each wide crest as abscissæ and the measured depths on the crest as ordinates.

Plate VII. is a plot of the experiments on the 4-inch crest reduced in size from the original; the corrections are plotted at five times their true size. This plot is typical of all of the others.

From an examination of the diagram it may be seen that there is a depth on the weir at which no correction is necessary; below this depth the correction is minus, *i. e.*, the depth on the wide crest is the greater one, and above it the correction is plus.

The curve represented by a dotted line is a hyperbola of a form which was found to agree closely with the experiments upon all of the crests.

The principal features of the hyperbola are as follows:

Its vertex at D, has an abscissa C, D, equal to $0.0942\ w$ (w being the width of the crest), and an ordinate A, C, equal to $0.807\ w$. Its axis is parallel with the axis of abscissæ. The depth A, B, at which there is no correction, is twice A, C, and equals $1.614\ w$. Transforming the equation of the hyperbola so as to make the required correction the first term, we have the following general formula for correcting the depth on a wide crest:

$$C = 0.2016\ \sqrt{y^2 + 0.2146\ w^2} - 0.1876\ w,$$

in which,

C is a correction to be added, algebraically, to the depth on the wide crest to obtain the depth on a sharp crest which will pass an equal volume of water.

w is the width of the crest.

y is the difference between $0.807\ w$ and the depth on the crest.

By referring to the diagram it will be seen that the dotted line representing the formula does not agree exactly with the experiments, and upon comparing the plots of the different crests, similar small disagreements were found which followed somewhat regular laws.

It was not thought best to make the formula more complicated in order to follow the small deviations from the hyperbola, since the error in the discharge caused by neglecting them is generally but a small fraction of one per cent.

They have, however, been tabulated as shown on page 88, and the secondary correction there given may be used in addition to the correction given by the main formula when the most accurate results are desired.

TABLE XXI.

SECONDARY CORRECTIONS TO BE ADDED ALGEBRAICALLY TO THE VALUE OF C AS FOUND BY THE FORMULA $C = 0.2016\sqrt{y^2 + 0.2146\,w^2} - 0.1876\,w$.

Width of Crest.	DEPTH ON WIDE CREST.					
	Depth $=0.3\,w$.	Depth $=0.6\,w$.	Depth $=0.9\,w$.	Depth $=1.2\,w$.	Depth $=1.5\,w$.	Depth $=1.8\,w$.
Inches.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.
1	—.0010	—.0016	—.0016	—.0014	—.0015	—.0006
2	—.0015	—.0021	—.0012	—.0009	—.0010	—.0001
3	—.0017	—.0020	—.0004	—.0004	—.0005	+.0004
4	—.0012	—.0010	+.0004	+.0002	.0000	+.0009
5	—.0006	+.0003	+.0012	+.0009	+.0006	+.0015
6	—.0001	+.0012	+.0019	+.0016	+.0011	+.0020
7	+.0002	+.0015	+.0024	+.0022	+.0017	+.0025
8	+.0004	+.0017	+.0028	+.0029	+.0022	+.0031
9	+.0006	+.0018	+.0031	+.0034	+.0027	+.0036
10	+.0007	+.0019	+.0032	+.0038	+.0032	+.0041
12	+.0008	+.0020	+.0034	+.0045	+.0042	+.0052
18	+.0010	+.0024	+.0040	+.0057	+.0063	+.0081
24	+.0011	+.0026	+.0044	+.0065	+.0077	+.0103
36	+.0013	+.0030	+.0050	+.0072	+.0098	+.0128

NOTE.—The broken lines represent the limits of the experiments.

PLATE VII.

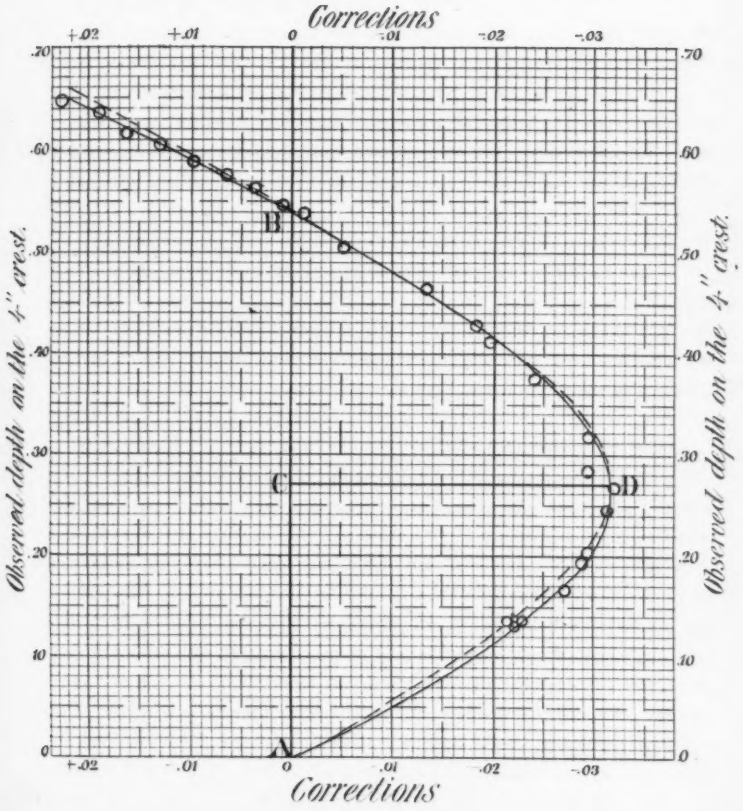
DIAGRAM SHOWING THE RESULTS OF EXPERIMENTS UPON THE DISCHARGE OVER A CREST 4 INCHES WIDE.

NOTE.—Each circle shows the result of an experiment.

The curve drawn with a broken line is a hyperbola and is constructed from the formula

$$C=0.2016\sqrt{y^2+0.2146w^2}-0.1876w.$$

The curve drawn with a full line was obtained by modifying the hyperbola by the secondary corrections given in Table XXI.



It was by using this secondary correction that the curve on the diagram, shown by a full line, was drawn.

The reasons why the corrections are successively minus, zero and plus as the depth on the weir increases, became apparent from observations made during the experiments, and are illustrated by Plate VIII., Figs. 1, 2, 3 and 4.

Fig. 1 shows the well-known form of the sheet passing a sharp crested weir, in which the bottom contraction causes the sheet to rise above the level of the crest for a distance *a b*.

Fig. 2 shows the case of the minus correction which occurs when the crest is wider than *a b*. The dotted lines show the form of the sheet passing the sharp crested weir as given in Fig. 1.

The bottom contraction of the flowing sheet remains with the wide crest nearly the same as with the sharp one, except that the space beneath the sheet is filled with water instead of air.

The sheet which would pass between the points *b c*, were the crest removed, is deflected, and assumes the form indicated by the full lines. Owing to the retardation of the flow caused by this deflection, additional depth on the weir is required to pass the same volume of water.

Fig. 3 shows the case of the zero correction which occurs when the width of the crest very nearly equals *a b*. In this case the form of the sheet is not changed appreciably, and the discharge takes place under nearly the same circumstances as over a sharp crest, except that in the case of the wide crest there is water between the flowing sheet and the crest, while in the case of the sharp crest there is air under all parts of the sheet.

Fig. 4 shows a case in which the crest is somewhat narrower than *a b*. From such a crest the sheet may be detached, as shown by the dotted lines, or it may be attached as shown by the full lines. In the former case it is evident that the width of the crest has no effect upon the discharge; the latter case is the one in which the plus correction occurs, and it is occasioned in this way: The sheet has a tendency to rise to its natural position as shown by the dotted lines, but *c* being the only place, as the apparatus used was arranged, at which air can find access to the space between the sheet and the crest, it will only enter there when the upward tendency of the sheet is considerable. If the air does not enter, a partial vacuum is created in the space between the crest and the sheet, which diminishes the contraction and consequently

depresses the whole sheet, thus requiring an addition to the measured depth before applying the general weir formula; this correction, however, is applicable only when the water adheres to the crest.

The depth at which the sheet becomes detached from the crest has been found by observation to vary from 1.614 to 1.95 times its width. Under ordinary circumstances, when the condition of the flow with regard to the presence of air under the sheet is not observed, this correction may be used until the depth over the weir is 1.75 times the width of the crest.

The results of all of the experiments are given in Table XXII.

The degree of accuracy reached may be seen from an examination of column 7.

To sum up the results given in this column, it may be stated that out of 92 experiments, 79 showed differences not exceeding 0.001 of a foot, 10 showed differences varying from 0.0011 to 0.0035 feet.

The three remaining ones were known to be defective and showed differences varying from 0.0042 to 0.0079 feet.

TABLE XXII.

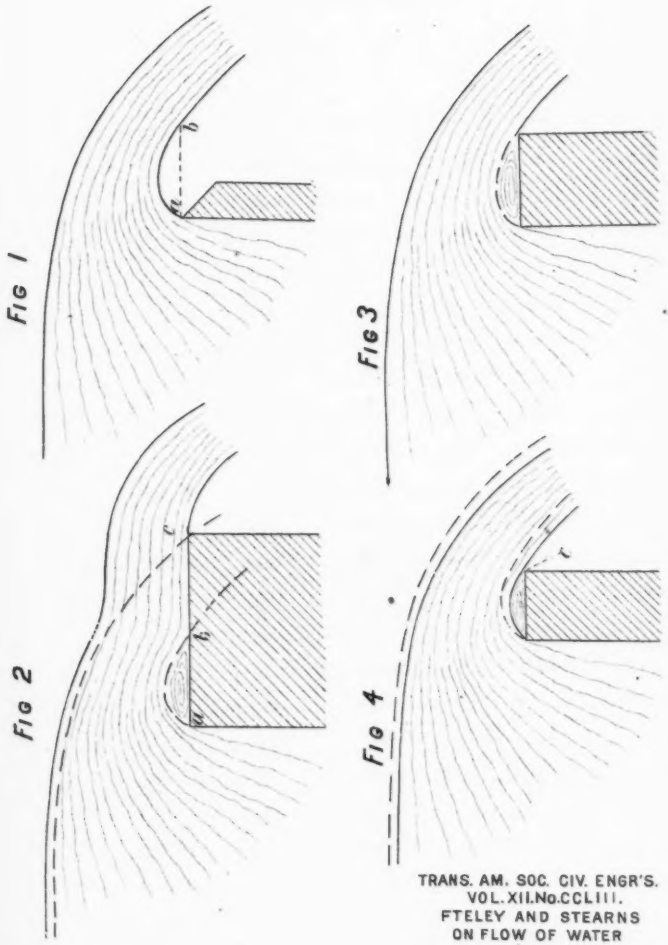
EXPERIMENTS UPON THE FLOW OF WATER OVER WEIRS WITH WIDE CRESTS.

CREST 2 INCHES WIDE.

1 Number of the Experiment.	2 Date of the Experiment. 1877.	3 Observed Depth on the Wide Crest.	4 Correction Calculated from the Formula for Wide Crests, including the Secondary Correction.	5 Depth on the Sharp Crest obtained by Correcting the Depth on the Wide Crest, given in Column 3.	6 Depth on the Sharp Crest as Observed.	7 Difference of the Calculated from the Observed Depth on the Sharp Crest.
		Feet.	Feet.	Feet.	Feet.	Feet.
1	March 16.	0.1158	— 0.0172	0.0986	0.0986	0.0000
2	" "	0.1360	— 0.0172	0.1188	0.1190	— 0.0002
3	" "	0.1722	— 0.0149	0.1573	0.1572	+ 0.0001
4	" "	0.2047	— 0.0111	0.1936	0.1939	— 0.0003
5	" "	0.2363	— 0.0065	0.2298	0.2297	+ 0.0001
6	" "	0.2486	— 0.0045	0.2441	0.2440	+ 0.0001
7	" "	0.2926	+ 0.0039	0.2965	0.2966	— 0.0001

PLATE VIII.

SKETCHES SHOWING VARIOUS CASES OF FLOW OF WATER OVER WIDE CRESTS.



TRANS. AM. SOC. CIV. ENGR'S.
VOL. XII. No. CCLIII.
FTELEY AND STEARNS
ON FLOW OF WATER

TABLE XXII.—(*Continued.*)

CREST 3 INCHES WIDE.

1	2	3	4	5	6	7
8	March 10	0.1307	— 0.0216	0.1091	0.1088	+ 0.0003
9	" 13	0.1329	— 0.0218	0.1111	0.1112	— 0.0001
10	" 9	0.1621	— 0.0241	0.1380	0.1375	+ 0.0005
11	" 14	0.1974	— 0.0244	0.1730	0.1728	+ 0.0002
12	" 7	0.2111	— 0.0240	0.1871	0.1882	— 0.0011
13	" 7	0.2119	— 0.0240	0.1879	0.1882	— 0.0003
14	" 14	0.2348	— 0.0229	0.2119	0.2122	— 0.0003
15	" 8	0.2694	— 0.0202	0.2492	0.2491	+ 0.0001
16	" 12	0.2978	— 0.0169	0.2809	0.2810	— 0.0001
17	" 7	0.3227	— 0.0135	0.3092	0.3086	+ 0.0006
18	" 8	0.3512	— 0.0093	0.3419	0.3427	— 0.0008
19	" 14	0.3561	— 0.0085	0.3476	0.3480	— 0.0004
20	" 8	0.3891	— 0.0030	0.3861	0.3870	— 0.0009
21	" 14	0.3928	— 0.0023	0.3905	0.3912	— 0.0007
22	" 14	0.3985	— 0.0013	0.3972	0.3981	— 0.0009
23	" 13	0.4053	0.0000	0.4053	0.4062	— 0.0009
24	" 14	0.4072	+ 0.0004	0.4076	0.4081	— 0.0005
25	" 14	0.4143	+ 0.0017	0.4160	0.4167	— 0.0007
26	" 14	0.4256	+ 0.0039	0.4295	0.4293	+ 0.0002
27	" 14	0.4484	+ 0.0084	0.4568	0.4568	0.0000
28	" 14	0.4619	+ 0.0111	0.4730	0.4730	0.0000

TABLE XXII.—(*Continued.*)

CREST 4 INCHES WIDE.

1	2	3	4	5	6	7
29	March 10	0.1318	— 0.0222	0.1096	0.1095	+ 0.0001
30	" 13	0.1331	— 0.0224	0.1107	0.1114	— 0.0007
31	" 14	0.1368	— 0.0228	0.1140	0.1139	+ 0.0001
32	" 9	0.1659	— 0.0263	0.1396	0.1390	+ 0.0006
33	" 15	0.1929	— 0.0289	0.1640	0.1642	— 0.0002
34	" 14	0.2036	— 0.0297	0.1739	0.1741	— 0.0002
35	" 14	0.2445	— 0.0313	0.2132	0.2136	— 0.0004
36	" 15	0.2665	— 0.0313	0.2352	0.2349	+ 0.0003
37	" 8	0.2814	— 0.0310	0.2504	0.2520	— 0.0016
38	" 12	0.3124	— 0.0297	0.2827	0.2830	— 0.0003
39	" 14	0.3745	— 0.0245	0.3500	0.3504	— 0.0004
40	" 8	0.4106	— 0.0201	0.3905	0.3909	— 0.0004
41	" 13	0.4273	— 0.0178	0.4095	0.4093	+ 0.0002
42	" 12	0.4616	— 0.0127	0.4489	0.4485	+ 0.0004
43	" 9	0.5034	— 0.0059	0.4975	0.4983	— 0.0008
44	" 12	0.5382	+ 0.0002	0.5384	0.5369	+ 0.0015
45	" 14	0.5447	+ 0.0014	0.5461	0.5456	+ 0.0005
46	" 14	0.5612	+ 0.0045	0.5657	0.5650	+ 0.0007
47	" 14	0.5750	+ 0.0072	0.5822	0.5815	+ 0.0007
48	" 12	0.5761	+ 0.0074	0.5835	0.5828	+ 0.0007
49	" 14	0.5912	+ 0.0103	0.6015	0.6012	+ 0.0003
50	" 14	0.6066	+ 0.0133	0.6199	0.6198	+ 0.0001
51	" 14	0.6168	+ 0.0153	0.6321	0.6329	— 0.0008
52	" 14	0.6368	+ 0.0194	0.6562	0.6558	+ 0.0004
53	" 14	0.6484	+ 0.0218	0.6702	0.6716	— 0.0014

TABLE XXII.—(*Continued.*)

CREST 6 INCHES WIDE.

1	2	3	4	5	6	7
54	March 10	0.1320	— 0.0220	0.1100	0.1092	+ 0.0008
55	" 13	0.1349	— 0.0224	0.1125	0.1115	+ 0.0010
56	" 9	0.1666	— 0.0270	0.1396	0.1394	+ 0.0002
57	" 7	0.2237	— 0.0343	0.1894	0.1896	— 0.0002
58	" 7	0.2237	— 0.0343	0.1894	0.1899	— 0.0005
59	" 8	0 2928	— 0.0409	0.2519	0.2519	0.0000
60	" 12	0.3255	— 0.0431	0.2824	0.2832	— 0.0008
61	" 16	0.3474	— 0.0442	0.3032	0.3023	+ 0.0009
62	" 7	0.3564	— 0.0446	0.3118	0.3121	— 0.0003
63	" 8	0.3912	— 0.0453	0.3459	0.3456	+ 0.0003
64	" 8	0.4352	— 0.0448	0.3904	0.3904	0.0000
65	" 12	0.4913	— 0 0419	0.4494	0.4492	+ 0.0002
66	" 9	0.5371	— 0.0381	0.4990	0.4984	+ 0.0006
67	" 12	0.5717	— 0.0344	0.5373	0.5373	0.0000
68	" 12	0.6131	— 0.0292	0.5839	0.5838	+ 0.0001
69	" 14	0.6617	— 0.0225	0.6392	0 6386	+ 0.0006
70	" 7	0.6929	— 0.0178	0.6751	0.6739	+ 0.0012
71	" 16	0.7026	— 0,0164	0.6862	0.6855	+ 0.0007
72	" 16	0.7440	— 0.0096	0.7344	0.7347	— 0.0003
73	" 9	0.7760	— 0.0042	0.7718	0.7716	+ 0.0002
74	" 16	0.7907	— 0 0017	0.7890	0.7892	— 0.0002
*75	" 10	0.8075	+ 0.0013	0.8088	0.8009	+ 0.0079

* See Experiments 91 and 92.

TABLE XXII.—(*Continued.*)

CREST 10 INCHES WIDE.

1	2	3	4	5	6	7
76	March 10	0.1352	— 0.0226	0.1126	0.1120	+ 0.0006
77	" 13	0.1394	— 0.0233	0.1161	0.1153	+ 0.0008
78	" 9	0.1712	0.0283	0.1429	0.1449	— 0.0020
79	" 15	0.2872	— 0.0455	0.2417	0.2422	— 0.0005
80	" 15	0.3403	— 0 0525	0.2878	0.2877	+ 0.0001
81	" 12	0.3449	— 0 0531	0.2918	0.2924	— 0.0006
82	" 13	0.4907	— 0.0684	0.4223	0.4224	— 0.0001
83	" 12	0.5350	— 0.0716	0.4634	0.4634	0.0000
84	" 9	0.5861	— 0.0741	0.5120	0.5155	— 0.0035
85	" 12	0.6307	— 0.0754	0.5553	0.5544	+ 0.0009
86	" 12	0.6794	— 0.0756	0.6038	0.6017	+ 0.0021
87	" 14	0.7323	— 0.0744	0.6579	0.6581	— 0.0002
88	" 16	0.7776	— 0.0723	0.7053	0.7061	— 0.0008
89	" 16	0.8222	— 0.0693	0.7529	0.7561	— 0.0032
90	" 16	0.8752	— 0.0648	0 8104	0.8129	— 0.0025
*91	" 10	0.8913	— 0.0632	0.8281	0.8239	+ 0.0042
*92	" 10	0.8941	— 0.0629	0.8312	0.8250	+ 0.0062

* Experiments 75, 91 and 92 were made in one series. A marked variation in depths on the weir occurred from time to time, caused probably by the smallness of the opening into the coffer dam, perhaps further diminished by some floating ice or wood. A second opening was made before the experiments were continued.

It may be stated that although the results given in the table are for crests of even inches in width, yet the experiments were made upon crests which varied a little from these dimensions.

The widths also increased slightly from day to day, owing to the swelling of the wood. A correction was made for these differences before constructing the tables.

The experiments were made with depths on the weir varying from 0.12 to 0.89 feet, and with crests varying in width from 2 to 10 inches.

From the accuracy with which the formula (including the secondary correction) agrees with this range of experiments, it seems probable that it may be applied without much error to crests not exceeding two or perhaps three feet in width, and to large depths on the weir, provided the depth of the channel is sufficient to prevent a large velocity of approach.

It is not probable that the formula will apply to depths on the weir under 0.10 feet, nor to crests less than one inch wide.

For convenience in practical use, Table XXIII., page 96, has been calculated, by using both the formula and the secondary correction. It gives the correction for nine widths of crests varying from one inch to two feet, and for depths on the weir from 0.05 to 1.50 feet; the correction for intermediate depths on the weir may be obtained by direct proportion, or more accurately by constructing from the data given in the table, a curve similar to that shown for the 4-inch crest on the diagram in Plate VII.

TABLE XXIII.

CORRECTIONS to be added algebraically to depths on a wide crest, to obtain the depths on a sharp crested weir, which will pass an equal volume of water.

Depth on Wide Crest.	WIDTH OF CREST.								
	1 Inch.	2 Inches.	3 Inches.	4 Inches.	6 Inches.	8 Inches.	10 Inches.	12 Inches.	2 Feet.
Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.
0.00	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
0.05	—.0087	—.0097	—.0097	—.0093	—.0088	—.0086	—.0086	—.0086	—.0087
0.10	—.0068	—.0163	—.0177	—.0176	—.0170	—.0168	—.0169	—.0169	—.0172
0.15	+.0022	—.0166	—.0233	—.0244	—.0246	—.0248	—.0249	—.0251	—.0256
0.20		—.0118	—.0243	—.0294	—.0314	—.0323	—.0328	—.0331	—.0340
0.25		—.0043	—.0219	—.0313	—.0370	—.0393	—.0402	—.0408	—.0423
0.30		+.0055	—.0167	—.0304	—.0414	—.0455	—.0473	—.0483	—.0504
0.35			—.0095	—.0270	—.0443	—.0509	—.0536	—.0554	—.0585
0.40			—.0010	—.0215	—.0453	—.0552	—.0597	—.0621	—.0665
0.45			+.0087	—.0145	—.0442	—.0581	—.0648	—.0683	—.0743
0.50				—.0065	—.0413	—.0600	—.0692	—.0740	—.0820
0.55				+.0024	—.0368	—.0622	—.0725	—.0791	—.0896
0.60				+.0120	—.0310	—.0587	—.0746	—.0833	—.0971
0.65				+.0221	—.0242	—.0558	—.0757	—.0867	—.1044
0.70					—.0168	—.0517	—.0753	—.0892	—.1116
0.75					—.0087	—.0469	—.0737	—.0907	—.1184
0.80					—.0001	—.0405	—.0709	—.0912	—.1250
0.85					+.0091	—.0338	—.0671	—.0906	—.1314
0.90					+.0186	—.0266	—.0623	—.0889	—.1375
0.95					+.0285	—.0190	—.0567	—.0861	—.1434
1.00						—.0109	—.0504	—.0825	—.1491
1.10						+.0066	—.0366	—.0729	—.1594
1.20						+.0253	—.0212	—.0606	—.1680
1.30						+.0447	—.0045	—.0467	—.1750
1.40							+.0133	—.0316	—.1802
1.50							+.0319	—.0154	—.1835

NOTE.—Use table only when sheet adheres to crest.

EXPERIMENTS ON THE FLOW OF WATER OVER A WEIR WHICH HAD THE UP-STREAM EDGE OF THE CREST ROUNDED.

Among the precautions to be observed in preparing a weir to which the common formulæ will apply, it is required that the up-stream edge shall be sharp, since it is stated that any appreciable rounding will increase the discharge materially. The amount of increase in the discharge seems never to have been determined, and as it is difficult to maintain a very sharp edge on a wooden crest during long-continued measurements, the following described experiments were made to ascertain the effect of the rounded edge. It was impracticable to make the experiments with a very small rounding of the edge, such as might occur in practice, and consequently the experiments were made upon rounded edges of larger radii, with the view of deducing from them the effect of smaller ones.

The general arrangement of the apparatus was the same as during the experiments upon wide crests; the rounded edges of $\frac{1}{4}$, $\frac{1}{2}$ and 1 inch radius, were made by attaching pieces to the face of the weir, as shown by Figures 6, 7 and 8, Plate I.; their position when attached to the weir is shown by the dotted lines in Fig. 5. The sides of the pieces which adjoined the weir, and the face of the weir, were carefully planed, and when fitted together adhered so firmly that no fastening was required.

The experiments were made in the same general manner and with nearly the same care as the experiments upon the wide crests.

One series of experiments was made with the rounded corners attached to the weir, which had a level crest 0.035 feet wide, and another when they were attached to a level crest 4 inches wide.

In the first series, when the depth on the weir was above a certain limit, the sheet of water lifted from the level crest and from a portion of the rounded edge about in the manner shown in Plate IX.

This limit was reached with the rounded edges of $\frac{1}{4}$, $\frac{1}{2}$ and 1 inch radii when the depths on the weir were 0.17, 0.26 and 0.45 feet respectively.

The form of the sheet, as given in the figure, shows that the addition of a rounded edge has practically the same effect as a lowering of the level of the barrier over which the water flows. The point, *a*, at which the sheet begins to separate from the crest, is approximately the point

at which the bottom curve of the sheet and the curve of the crest become tangent. This curve of the sheet so near its beginning is known to be very nearly constant in direction for all except the smaller depths on the weir. This fact indicates that the position on the crest of the point of separation, a, is nearly constant. With crests of large radius the portion of the curve below a would probably modify appreciably the form of the bottom curve of the sheet, but with crests of small radius it is doubtful if this portion of the curve has an important effect upon the flow.

The experiments in which the sheet lifted in the manner shown in the sketch, were made only upon the crests of $\frac{1}{4}$ and $\frac{1}{2}$-inch radius; they showed that the only correction necessary before using the formula for a sharp-crested weir was to add $\frac{7}{10}$ of the radius of the crest to the measured depth on the weir, *i.e.*, the discharge over a rounded crest is practically the same as over a sharp crest placed lower to the amount of $\frac{7}{10}$ of the radius of the rounded crest; *i.e.*, approximately, at the level of the point a, in the figure.

The results given may be expressed by the formula

$$C = 0.7\ R,$$

in which C is the correction to be added to the measured depth on a rounded crest to find the depth on a sharp crest which will pass an equal volume of water. R is the radius of the crest.

This formula is limited to crests having a radius of less than $\frac{1}{2}$ of an inch and to depths on the weir which are sufficiently large to cause the sheet to lift from a portion of the crest, as shown on Plate IX.; it will probably become inaccurate when the depth on the weir is less than 0.15 feet.

From the point where the sheet of water ceased to be lifted from the crest, the term C of the formula was shown by the experiments to diminish uniformly to zero, when the depth on the weir was about 0.09 feet.

Table XXIV. contains the record of all the experiments of this series, and shows the comparison between the actual results and those obtained by the formula $C = 0.7\ R$ in all cases where it is applicable.

The differences resulting from this comparison may be seen in column 7. The largest difference, 0.0042 feet, occurs in an experiment made at a later date than the others, after the steel edge had been fastened to the weir and generally under more unfavorable conditions than the others. The next largest difference is 0.0032 feet. The seven remaining differences do not in any case exceed 0.001 feet.

PLATE IX.

SKETCH SHOWING WATER FLOWING OVER A ROUNDED CREST.

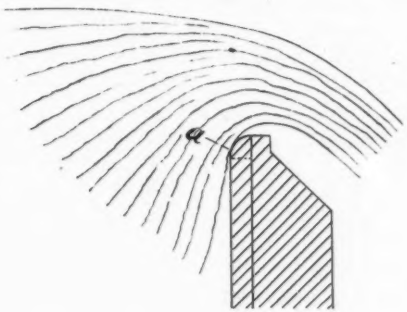


TABLE XXIV.

Experiments upon the Flow of Water over Rounded Crests.

1	2	3	4	5	6	7
Number of the Experiment.	Date. 1877.	Observed Depth on the Top of the Rounded Crest.	Observed Depth on the Sharp Crest.	Correction obtained from Experiment by deducting the Depth in Col. 3 from the Depth in Col. 4.	Correction Calculated by the Formula $C=0.7R$.	Difference of the Observed from the Calculated Correction.
Experiments upon the Crest of ¼-inch Radius						
		Feet.	Feet.	Feet.	Feet.	Feet.
1	March 14	0.1096	0.1159	0.0063		
2	" 15	0.1524	0.1662	0.0138		
3	" 15	0.2723	0.2837	0.0114	0.0146	—0.0032
4	" 14	0.3408	0.3556	0.0148	0.0146	+0.0002
5	" 15	0.3953	0.4096	0.0143	0.0146	—0.0003
6	" 16	0.4742	0.4889	0.0147	0.0146	+0.0001
Experiments upon the Crest of ½-inch Radius.						
7	March 14	0.1118	0.1158	0.0040		
8	" 15	0.1535	0.1663	0.0128		
9	" 14	0.1955	0.2166	0.0211		
10	" 15	0.2547	0.2835	0.0288	0.0292	—0.0004
11	" 14	0.3273	0.3555	0.0282	0.0292	—0.0010
12	" 15	0.3813	0.4096	0.0283	0.0292	—0.0009
13	" 16	0.4594	0.4889	0.0295	0.0292	+0.0003
14	April 2	0.6455	0.6789	0.0334	0.0292	+0.0042
Experiments upon the Crest of 1-inch Radius.						
15	March 14	0.1127	0.1157	0.0030		
16	" 15	0.1569	0.1665	0.0096		
17	" 14	0.1986	0.2166	0.0180		
18	" 15	0.2553	0.2833	0.0280		
19	" 14	0.3164	0.3555	0.0391		
20	" 15	0.3656	0.4096	0.0440		
21	" 16	0.4371	0.4888	0 0517		

The second series of experiments, in which the water flowed over a rounded approach attached to a crest four inches wide, was made to ascertain whether or not the formulæ already given for wide and rounded crests were applicable when both conditions existed at the same weir, and it was found that they were not. It was ascertained, however, that by using the original formula for the wide crest, and a modified formula for the effect of the rounded approach, correct results could be obtained. The modified formula is $C = 0.41\ R$, and varies only from that previously given in the value of the co-efficient. In measuring the width of a crest the width of the rounded approach should be included.

The modified formula has the same limits as the original one, both in regard to the radii of the rounded approach and to the depths on the weir, *i.e.*, it is limited to radii not exceeding $\frac{1}{2}$ of an inch and to depths on the weir not less than 0.17 and 0.26 feet respectively, in the cases of the $\frac{1}{4}$ and $\frac{1}{2}$ inch crests. It is also limited to crests of about the width experimented upon or from 4 to 5 inches.

This series of experiments was not made with a view of determining any strict formula, but rather to ascertain to what extent a measurement of the discharge over a plank slightly rounded at the up-stream corner could be relied upon. Seventeen experiments of this class were made, but they are not deemed of sufficient importance to be given in detail.

EXPERIMENTS UPON THE DISCHARGE OF WATER OVER A SUBMERGED WEIR.

This series concluded the experiments of 1877, and, on account of the necessity of removing the apparatus, it was made more hastily than the others, the entire series being completed in one day.

The apparatus is, with the exception of some additions for measuring the height of the water on the down-stream side of the weir, the same as that used for measuring the discharge over a sharp-crested weir, as shown on Plate I. and described on pages 52 to 53. The level of the water on the down-stream side of the weir was controlled by stop-planks placed in the first dam below the weir. Owing to the varying pressure at different points on the down-stream side, caused by the large velocity of the water passing the weir, the place where the head is taken becomes a matter of considerable importance. In these experiments it

was thought best to take the head at a place which would represent fairly the height of the comparatively still water below the weir. With this in view, a pipe leading from a hook-gauge pail was terminated at the bottom of the channel near its side, 6 feet from the weir, and to protect its end from lateral and vertical currents it was inserted in a hole in the side of an inverted trough, *R* (Plate I., Figs. 1 and 2), 2 feet long and 4 inches square in section, the sides of the trough being parallel with the side of the channel.

It will be noticed by reference to the same plate that the channel below is wider than the weir, consequently the pipe ending near the side of the channel was not exposed directly to the force of the current of water passing the weir.

The method of making these experiments was similar to that employed for other modifications of the weir. The constant volume of water was made to pass over the sharp-crested weir when the discharge was free, and also over the same weir when the water on the down-stream side stood above the level of its crest. When in the latter condition the height of the water on the up-stream and down-stream sides of the weir was measured.

It was not found practicable to represent the results of this modification of the weir by a formula for correcting the depth on the weir, as it has been done in previous instances; therefore the following formula, which gives directly the volume discharged, was adopted for calculating the results:

$$Q = c\,l\left(d + \frac{d'}{2}\right)\sqrt{h}$$

in which

Q is the quantity of water discharged in cubic feet per second;

c is a co-efficient deduced from experiment, which was found to vary with the ratio $\frac{d'}{d}$;

l is the length of the weir;

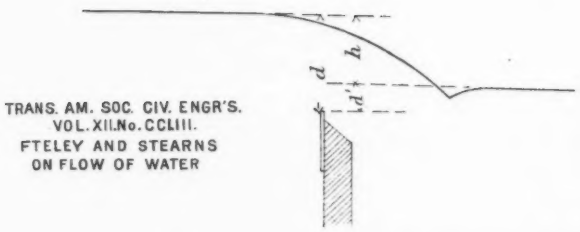
d is the depth on the weir measured from the still water on the up-stream side;

d' is the depth to which the weir is submerged measured from the still water on the down-stream side;

h is the difference between the levels of the water on the up-stream and down-stream sides of the weir, and consequently equals $d-d'$.

(See Plate X., Fig. 1.)

PLATE X. Fig. 1.



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ON FLOW OF WATER

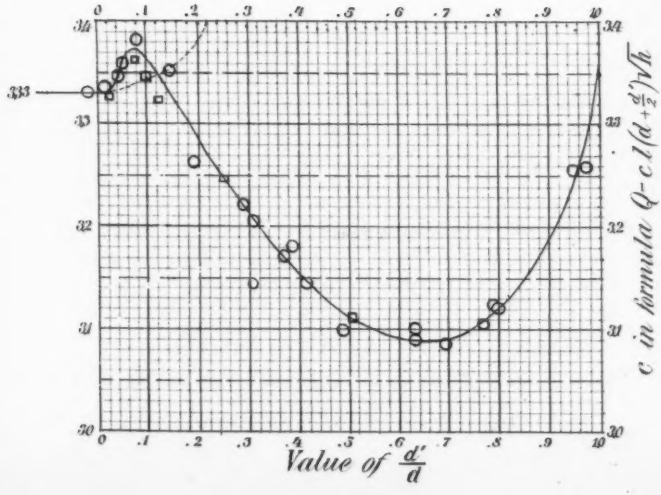
DIAGRAM ILLUSTRATING EXPERIMENTS UPON THE FLOW OF WATER OVER A SUBMERGED WEIR.

EXPLANATION OF FIG. 2.

Experiments made by the Authors shown thus ○
" " " Mr. J. B. Francis " □

The full curve is intended to show the mean results of the experiments. The dotted curve shows a series of coefficients which, when used in the formula for the submerged weir, furnish results identical with the formula $Q = 3.33\ L\ H^{\frac{3}{2}}$

Fig. 2.



From the above formula

$$c = \frac{Q}{l\left(d + \frac{d'}{2}\right)\sqrt{h}}$$

Q is the only quantity in the second term of this equation which is not the result of actual measurement, and its value is calculated from the measurements at the weir, when not submerged, by Mr. Francis' well-known formula $Q = 3.33\ LH^{\frac{3}{2}}$.

By this method the value of c was determined from each experiment; it proved to be a variable quantity; a further examination showed that it varied with the proportion o the depth on the weir which was submerged, a ratio represented by $\frac{d'}{d}$.

Mr. Francis has placed on record a series of "experiments on the effect produced on the flow of water over weirs, by the ight of the water on the down-stream side,"* which were made in a manner almost identical with our own; these experiments are used in connection with those made by us to determine the value of the co-efficients of the formula.

The results of both series of experiments are shown in Table XXV., and also on Plate X. On the latter the ordinates represent the co-efficients of the formula for a submerged weir and the abscissæ the proportion of the total depth on the weir, which is submerged, a ratio represented by $\frac{d'}{d}$. The circles show our own experiments, the squares those made by Mr. Francis. The curve drawn with a full line represents the co-efficients finally adopted for the varying values of $\frac{d'}{d}$. The short curve shown by the dotted line represents a series of co-efficients which, when used in the formula for a submerged weir, will give the same discharge as Mr. Francis' formula, $Q = 3.33\ LH^{\frac{3}{2}}$, disregarding in the latter case the height of the water on the down-stream side of the weir.

In the cases where the full line is above the dotted one the flow over the weir is facilitated by the slightly increased height of the water on the down-stream side. The proximity of the two curves until $\frac{d'}{d} = 0.15$ shows that until this limit is reached the fact that the weir is submerged need not be considered, provided an error of one per cent. is

* Lowell Hyraulic Experiments, p. 102.

admissible, and provided air has free access to the space beneath the sheet until $\frac{d'}{d} = 0.08$.

The curve of co-efficients has some peculiarities of form which the circumstances of the experiments can account for in part. During experiments 2 and 1′, although the water on the down-stream side was slightly above the crest of the weir, yet the air remained under the sheet and its form was not perceptibly changed. The average result of these two experiments does not show any appreciable change in the lischarge caused by raising the water on the down-stream side to this level, and in consequence the two curves on the diagram correspond to this point.

During Experiments 3, 4 and 5 the air remained under the sheet but a portion of the time, passing in and out with each oscillation of the water below the weir. The short piece of the curve between the points where $\frac{d'}{d} = 0.02$ and 0.08 represents this condition of the flow and establishes a connection between the first and last parts of the curve; the first representing the co-efficients in cases where the air remains permanently under the sheet, the last and longest part representing the co-efficients when no air remains.

In this last part it will be seen that the co-efficients decrease as the relative height of the water on the down-stream side becomes greater until $\frac{d'}{d} = 0.65$, when they again increase.

The cause of this marked depression in the curve is not definitely known.

The proportional differences between the result of each experiment and the formula may best be seen by referring to column 9 of the table.

The largest difference is in Experiment 9, $2\frac{1}{2}$ per cent., so much larger than any other that the experiment may be considered of no value. The next in size is in Experiment 7, a little less than one per cent. In this case a reference to the note-book showed that the height of the water on the down-stream side was lowering during the measurement, owing probably to the instability of some small stop-planks used to cause the smaller variations in the level of the water on the down-stream side of the weir.

Of the remaining 25 experiments in both series the differences are in all cases less than one per cent., and average 0.0028, or a little more than a fourth of one per cent.

TABLE XXV.

EXPERIMENTS ON THE FLOW OF WATER OVER A SUBMERGED WEIR, MADE IN THE SUDBURY RIVER CONDUIT, APRIL 3D, 1877. TEMPERATURE OF AIR NEAR THE WEIR, ABOUT 42°; TEMPERATURE OF WATER, ABOUT 37°.

1 Number of Experiment.	2 Depth on Submerged Weir measured from Still Water on the Up-stream Side. (d)	3 Depth to which the Weir was Submerged, measured from Still Water on the Down-stream Side. (d')	4 Proportion of Depth on the Weir which was Submerged. ($\frac{d'}{d}$)	5 Depth on Sharp-crested Weir with Free Sheet passing the same Volume of Water as the Submerged Weir. (H)	6 Co-efficient in formula $Q = cl\left(d + \frac{d'}{2}\right)\sqrt{h}$, as obtained from each experiment. (c)	7 Volume passing the Weir per foot length as calculated by the formula $Q = 3.33\ L\ H^{\frac{3}{2}}$, using the value of H given in column 5.	8 Volume passing the Weir per foot length as calculated by the formula $Q = cl\left(d + \frac{d'}{2}\right)\sqrt{h}$, using the value of c as found in Table XXVI, and the value of d and d' given in col. 2 and 3.	9 Proportional difference in volume as calculated by the two formulæ, the volume as given in column 7 being the standard.	10 REMARKS.
	Feet.	Feet.		Feet.	.....	Cubic Feet per sec.	Cubic Feet per sec.	.....	
1	0.3974	—0.0250	—0.063	0.3974	.....	.....	.....	.....	Air under the sheet.
2	0.5777	0.0098	0.017	0.5785	3.337	1.4652	1.4623	— 0 0020	Air under the sheet.
3	0.3960	0.0165	0.042	0.3972	3.347	0.8336	0.8334	— 0.0002	Air under the sheet a portion of the time.
4	0.6625	0.0330	0.050	0.6661	3.360	1.8103	1.8101	— 0.0001	
5	0.5743	0.0463	0.081	0.5793	3.382	1.4682	1.4638	— 0.0030	
6	0.5080	0.0735	0.145	0.5074	3.352	1.2035	1.1961	— 0 0062	No air under the sheet in this and succeeding experiments.
7	0.5922	0.1122	0.189	0.5788	3.265	1.4663	1.4799	+ 0.0093	
8	0.4157	0.1185	0.285	0.3974	3.222	0.8342	0.8346	+ 0 0005	
9	0.3251	0.0956	0.294	0.3053	3.144	0.5617	0.5748	+ 0.0233	
10	0.6098	0.1859	0.305	0.5790	3.206	1.4671	1.4689	+ 0.0012	
11	0.7165	0.2665	0.372	0.6655	3.171	1.8078	1.8064	— 0.0008	
12	0.5462	0.2095	0.383	0.5069	3.181	1.2017	1.1948	— 0.0057	
13	0.6328	0.2614	0.413	0.5782	3.147	1.4641	1.4650	+ 0.0006	
14	0.4504	0.2195	0.487	0.3973	3.098	0.8339	0.8390	+ 0.0061	
15	0.4857	0.3078	0.634	0.3972	3.090	0.8336	0.8335	— 0.0001	
16	0.8123	0.5157	0.635	0.6650	3.098	1.8058	1.8006	— 0.0029	
17	0.7425	0.5186	0.698	0.5780	3.087	1.4633	1.4656	+ 0.0016	
18	0.5454	0.4186	0.767	0.3973	3.103	0.8339	0.8352	+ 0.0015	
19	0.4282	0.3373	0.788	0.3053	3.121	0.5617	0.5608	— 0.0016	
20	0.7193	0.5731	0.797	0.5062	3.118	1.1993	1.2001	+ 0.0007	
21	0.6246	0.5894	0.944	0.3053	3.257	0.5617	0.5585	— 0.0055	
22	0.8149	0.7947	0.975	0.3053	3.260	0.5617	0.5668	+ 0.0091	

EXPERIMENTS ON THE FLOW OF WATER OVER A SUBMERGED WEIR, MADE BY JAMES B. FRANCIS, AT LOWELL, MASS., NOVEMBER 17TH, 1848.

1′	0.8532	0.020	0.023	0.8525	3.327	2.6211	2.6255	+ 0.0017	Air under the sheet.
2′	0.8485	0.065	0.077	0.8525	3.361	2.6211	2.6293	+ 0.0031	No air under the sheet in these experiments.
3′	0.8522	0.085	0.100	0.8525	3.345	2.6211	2.6370	+ 0.0061	
4′	0.8571	0.105	0.123	0.8525	3.323	2.6211	2.6421	+ 0.0080	
5′	0.8820	0.220	0.249	0.8525	3.247	2.6211	2.6229	+ 0.0007	
6′	0.9704	0.490	0.505	0.8525	3.111	2.6211	2.6211	0.0000	

The variable co-efficient of the formula as represented by the curve is also shown in Table XXVI., for each 0.01 in value of the ratio $\frac{d'}{d}$. The co-efficients were established by measurement from the original diagram, a reduced copy of which is shown on Plate X.

TABLE XXVI.

CO-EFFICIENTS IN FORMULA $Q = c\,l\left(d + \frac{d'}{2}\right)\sqrt{h}$ CORRESPONDING TO EACH ONE-HUNDREDTH IN VALUE OF THE RATIO $\frac{d'}{d}$.

$\frac{d'}{d}$	Co-efficient.	$\frac{d'}{d}$	Co-efficient.	$\frac{d'}{d}$	Co-efficient.	$\frac{d'}{d}$	Co-efficient.
0.01	3.330	0.26	3.241	0.51	3.110	0.76	3.105
.02	3.331	.27	3.234	.52	3.107	.77	3.109
.03	3.335	.28	3.227	.53	3.104	.78	3.113
.04	3.343	.29	3.220	.54	3.102	.79	3.117
.05	3.360	.30	3.214	.55	3.100	.80	3.122
.06	3 368	.31	3.207	.56	3.098	.81	3.127
.07	3.371	.32	3.201	.57	3.096	.82	3.131
.08	3.372	.33	3.194	.58	3.095	.83	3.137
.09	3.370	.34	3.188	.59	3.093	.84	3.143
.10	3.365	.35	3.182	.60	3.092	.85	3.150
.11	3.359	.36	3.176	.61	3.091	.86	3.156
.12	3.352	.37	3.170	.62	3.090	.87	3.164
.13	3.343	.38	3.165	.63	3.090	.88	3.172
.14	3.335	.39	3.159	.64	3.089	.89	3.181
.15	3.327	.40	3.155	.65	3.089	.90	3.190
.16	3.318	.41	3.150	.66	3.089	.91	3.200
.17	3.310	.42	3.145	.67	3.090	.92	3.209
.18	3.302	.43	3.140	.68	3.090	.93	3.221
.19	3.294	.44	3.135	.69	3.091	.94	3.233
.20	3.286	.45	3.131	.70	3.092	.95	3.247
.21	3.278	.46	3.127	.71	3.093	.96	3.262
.22	3.271	.47	3.123	.72	3.095	.97	3.280
.23	3.264	.48	3.119	.73	3.097	.98	3.300
.24	3.256	.49	3.116	.74	3.099	.99	3.325
.25	3.249	.50	3.113	.75	3.102	1.00	3.360

Limits of the Formula.

The quantity of water passing the weir during the experiments was, in all cases, calculated by Mr. Francis' formula, $Q = 3.33\, LH^{\frac{3}{2}}$, from measurements taken when the weir was not submerged. This formula was used in preference to the one given by us on page 82, because it is necessary for the simplicity of the formula for the submerged weir to base it upon a simple formula having a constant co-efficient, and the co-efficient 3.33 is probably more applicable to cases generally met with in practice than any other. In the case of depths on the weir to which the co-efficient 3.33 will not apply, the discharge as given by the formula for the submerged weir will be in error to nearly the same extent, and may be corrected by the table of percentages given on page 85.

Aside from the corrections just referred to, the absolute depth on the weir seems to have no influence on the co-efficient which varies, as far as can be determined from the experiments, only with the ratio $\frac{d'}{d}$; it may therefore be inferred that the formula will apply without much error to depths much larger than those experimented upon.

The formula will be inapplicable to values of $\frac{d'}{d}$ less than 0.08, unless air has free access to the space beneath the sheet. The correction for the effect of velocity of approach can be made by increasing the depth on the weir in the manner provided for weirs under ordinary conditions. The height of the water on the down-stream side should also be corrected for the velocity of the water leaving the weir. The latter correction is undoubtedly of great importance when the channel on the down-stream side is shallow, particularly when it is not wider than the weir; but since there is no data for making this correction, it becomes necessary to limit the use of the formula to cases in which such unfavorable conditions do not exist.

An instance of the error which might result from the use of this formula in a contracted channel is furnished by an extensive series of experiments* made by K. R. Bornemann.

The experiments were made in 1866–67–69–71 and '72. In 77 experiments, out of a total of 103, the depths on the weir were taken in such a manner that the experiments can be compared with our own.

The experiments were all made upon weirs which occupied the whole

* Der Civilingenieur. 1876.

width of the channel in which they were placed. The depths on the weirs varied in the different experiments from 0.31 to 0.879 feet.

The series of 1866-67 and '69 were made upon weirs 3.72 feet long; the height of the weirs above the bottom of the channel varied from 0.32 to 0.80 feet, and averaged 0.57 feet. The average depth on the weir was 0.63 feet. These experiments were 36 in number, and the average co-efficient of our formula deduced from them was 4.065, or about 27 per cent. greater than that deduced from our experiments.

The series of 1871 was made upon a weir 1.81 feet long. Its height varied from 0.33 to 0.79 feet, averaging 0.57 feet. The average depth on the weir was 0.51 feet, These experiments, 22 in number, furnished an average co-efficient of 3.50, which is about 10 per cent. greater than that deduced from our experiments.

The series of 1872 was made upon a weir 2.63 feet long. Its height varied from 0.40 to 0.80 feet, averaging 0.67 feet. The average depth on the weir was 0.50 feet. These experiments, 19 in number, furnished an average co-efficient of 3.54, which is about 11 per cent. greater than that deduced from our experiments.

The author has attributed the variation in the different series of his experiments to the different lengths of the weirs; but this seems very improbable; particularly, as all of the weirs were without end contractions, and it is disproved by the close comparison between our experiments and those of Mr. Francis; because, in the latter case, the weir was more than three times as long as in the former. We would attribute the variation of the experiments, among themselves and from our own, to the limited dimensions of the channel leading to and from the weir, which caused the water to approach and leave the weir with a large velocity.

The correction for velocity of approach was made by adding the theoretical head, due to the mean velocity of approach, to the measured depth on the weir, a correction which is not large enough, and no correction was made for the velocity with which the water left the weir.

The close accordance between the two series of experiments upon which our formula is based seems to warrant the conclusion that the formula will generally be reliable within one per cent., provided the channel on the down-stream side is sufficiently deep and wide; its dimensions are, however, much less important when the weir is but little submerged than under other circumstances.

In practice, this form of weir may be found useful in cases where, on account of the slight fall obtainable, it would be impossible to erect a weir of the ordinary form. In Experiment 22, the total fall was about one-fourth of an inch, and in Experiment 21 it was less than one-half of an inch, showing that a small fall is sufficient for a reliable measurement.

EXPERIMENTS TO DETERMINE THE EFFECT OF END CONTRACTION UPON THE FLOW OF WATER OVER WEIRS.

These experiments were made on March 21st and 22d, 1878. The apparatus was the same as that used during the experiments upon velocity of approach, as shown on Plate II., and described on pages 6 to 9.

In making the experiments, a constant quantity of water was made to flow, first over a weir 5 feet long, without end contractions, and afterwards over the same weir shortened with pieces, at one or both ends, to cause end contractions. Six different forms of weirs were used, one without and five with end contractions; the dimensions and conditions are given in the following table:

TABLE XXVII.

THE WIDTH OF THE CHANNEL WAS 5 FEET, AND ITS DEPTH BELOW THE CREST OF THE WEIR, 3.56 FEET.

Number designating the Form of the Weir.	Number of End Contractions.	Length of the Weir.	Distance from the side of the Channel to the end of the Weir.	
			North Side.	South Side.
		Feet.	Feet.	Feet.
1	0	5.0		
2	1	4.0	1.0	.0
3	1	4.0	.0	1.0
4	1	3.3	.0	1.7
5	2	3.0	1.0	1.0
6	2	2.3	1.0	1.7

The constant quantity of water discharged is computed from measurements taken at the first form of weir without end contractions, by using the formula $Q = 3.31\ LH^{\frac{3}{2}} + 0.007\ L$. (See page 82). Having the value

of Q thus determined, the same formula is used in the case of the other weirs to determine L, their effective length. The difference between this value of L, and the measured length of a weir, represents the effect of the end contraction.

In presenting the results, this effect is expressed as a fraction of the depth on the weir by using the formula

$$C = bH,$$

in which

C is a correction to be subtracted from the measured length of the weir for each end contraction ;

b is a co-efficient deduced from experiment ;

H is the depth on the weir corrected, if necessary, for the effect of velocity of approach.

The co-efficient, b, was found to vary to a marked extent with the depth on, and the form of the weir.

The experiments are given in Table XXVIII.

In this table most of the columns are sufficiently explained by their headings.

Column 8.—The co-efficients given in this column are, in the case of the weir without end contraction, taken from the table on page 12, and in the other cases from data furnished by the experiments given in Table III., pages 22 and 23.

Column 11.—The lengths of the weirs were measured at and 0.60 feet above the level of the crest. Two series of measurements were taken the first day of the experiments, at 9 A. M. and 2 P. M. ; one was taken the second day at 1.30 P.M. The measurements were compared with a steel standard, and they showed that the weirs continued to shorten during the experiments, owing to the swelling of the pieces used to cause end contraction.

Column 12.—The discharges in this column, which correspond to the experiments on the weir without end contractions, are calculated by the formula $Q = 3.31\, LH^{\frac{3}{2}} + 0.007\, L$. An average of the quantities thus obtained was generally assumed to be the discharge in the other experiments of the series ; but in several cases, when the measurements indicated that the volume passing the weir was not exactly a constant quantity, it was assumed that the discharge varied from the beginning to the end of the series in proportion to the time elapsing between the experiments.

Column 14.—The co-efficients given in this column are obtained by dividing the differences between the lengths of weir, as given in columns 11 and 13, by the depth on the weir.

A glance at this column shows that there is a large variation in the co-efficients; but on account of the arrangement of the experiments in the table, the nature and cause of the variation is not clearly indicated.

To show the results more clearly, the co-efficients have been plotted and are shown on Plate XI., which may be understood from the explanations accompanying it. Upon examining this plate it is noticeable that the co-efficients decrease quite rapidly as the depth on the weir increases, with all of the forms of weir experimented upon. The difference in the co-efficients for the different forms of weirs is also marked, although the conditions are in nearly all cases within the limits to which, in practice, a formula having a constant co-efficient is generally applied.

In Experiments 33 and 35 the lengths of the weirs were respectively 2.6 and 2.4 times the depths on the weirs.

In Experiments 40 and 45 the depth on the weir was 87, and in Experiments 51, 52 and 54 it was 95 per cent. of the distance from the side of the channel to the end of the weir.

The above are extreme cases, yet there is no indication that they are not governed by the same laws as the other experiments.

In the cases of Weirs Nos. 2, 3, and 4, the position of the co-efficients on the diagram do not coincide with the lines as exactly as in the other cases; this is due chiefly to the fact that any slight inaccuracy in the observations affects the co-efficient more when the weir is long and has but one end contraction.

The results of these experiments seemed somewhat peculiar, and it was thought advisable to examine the methods used so as to ascertain within what limits they are, probably, reliable.

By the method used, which appeared to be the only one available under the circumstances, it was necessary to make a marked change in the length of and depth on the weir, in order to produce the end contractions; this caused the experiments to depend to some extent upon the formula used for computing the discharge over the weir. In the formula given by us ($Q = 3.31\ L\ H^{\frac{3}{2}} + 0.007\ L$) the discharge does not vary exactly as $H^{\frac{3}{2}}$; while in the formulæ of Mr. Francis and several others it does.

Recalculating the experiments upon the basis that the discharge does vary directly with $H^{\frac{3}{2}}$, the following comparative results were obtained.

TABLE XXVIII.

Experiments to determine the effect of end contraction upon the flow of water over Weirs, made in the Sudbury Conduit, March 21st and 22d, 1878. Depth of channel below crest of weir, 3.56 feet. Temperature of water, 40°.

1 Number of the Series and of the Experiment.	2 Date and Time of the Experiment.	3 No. designating the form of the Weir as given in Table XXVII.	4 Number of End Contractions.	5 Observed Depth on the Weir taken 6 feet upstream.	6 Mean Velocity of Approach.	7 Theoretical Head due to Mean Velocity of Approach.	8 Co-efficient in Formula for Velocity of Approach.	9 Correction for the Effect of Velocity of Approach.	10 Depth on the Weir corrected for Velocity of Approach. (H)	11 Length of the Weir as measured.	12 Discharge over the Weir. (Q)	13 Effective Length of the Weir with End Contractions. (L)	14 Value of b in the Formula $C = b\,H$.	15 Remarks.
	1878.				Feet per Second.	Feet.		Feet.	Feet.	Feet.	Cu. Ft. per Sec.	Feet.		
Series I.	March 21.													
Exp. 1...	3.28 P.M.	1	0	0.1509	0.054	0.00005	1.51	0.0001	0.1510	5.0048	1.007			
" 2...	3.35 "	2	1	0.1761	0.054	0.00005	1.86	0.0001	0.1762	4.0062	1.007	3.9992	0.040	End contraction imperfect.
" 3...	3.42 "	3	1	0.1763	0.054	0.000 5	1.86	0 0001	0.1764	4.0064	1.007	3.9929	0.077	
" 4...	3.49 "	5	2	0.2155	0.053	0.00004	2.21	0.0001	0.2156	3.0080	1.007	2.9758	0.075	
" 5...	3.57 "	1	0	0.1509	0.054	0.00005	1.51	0.0001	0.1510	5.0048	1.007			
Series II.	March 21.													
Exp. 6...	4.07 P.M.	1	0	0.2303	0.098	0.00015	1.51	0.0002	0.2305	5.0044	1.868			
" 7...	4.15 "	2	1	0.2691	0.098	0.00015	1.85	0.0003	0.2694	4.0063	1.868	3.9762	0.112	
" 8...	4.19 "	3	1	0.2693	0.098	0.00015	1.85	0.0003	0.2696	4.0062	1.869	3.9732	0.122	
" 9...	4.24 "	5	2	0.3301	0.096	0.00014	2.20	0.0003	0.3304	3.0081	1.869	2.9405	0.102	
" 10...	4.30 "	1	0	0.2304	0.098	0.00015	1.51	0.0002	0.2306	5.0044	1.869			
Series III.	March 21.													
Exp. 11...	4.38 P.M.	1	0	0.3368	0.168	0.00044	1.50	0.0007	0.3375	5.0045	3.283			
" 12...	4.44 "	2	1	0.3942	0.166	0.00043	1.84	0.0008	0.3950	4.0063	3.283	3.9611	0.114	
" 13...	4.47 "	3	1	0.3944	0.166	0.00043	1.84	0.0008	0.3952	4.0061	3.283	3.9583	0.121	
" 14...	4.53 "	5	2	0.4843	0.162	0.00041	2.18	0.0009	0.4852	3.0080	3.284	2.9173	0.094	
" 15...	4.59 "	4	1	0.4498	0.164	0.00042	2.04	0.0009	0.4507	3.3110	3.284	3.2560	0.122	
" 16..	5.07 "	6	2	0.5824	0.158	0.00039	2.22	0.0009	0.5833	2.3132	3.284	2.2165	0.083	
" 17...	5.13 "	1	0	0.3369	0.168	0.00044	1.50	0.0007	0.3376	5.0045	3.285			
Series IV.	March 21.													
Exp. 18...	5.22 P.M.	1	0	0.4244	0.233	0.00084	1.49	0.0013	0.4257	5.0043	4.636			
" 19...	5.29 "	3	1	0.4978	0.228	0.00081	1.83	0.0015	0.4993	4.0058	4.636	3.9462	0.119	
" 20...	5.35 "	4	1	0.5678	0.224	0.00078	2.02	0.0016	0.5694	3.3107	4.637	3.2445	0.116	
" 21...	5.42 "	1	0	0.4245	0.233	0.00084	1.49	0.0013	0.4258	5.0043	4.637			
Series V.	March 22.													
Exp. 22...	8.44 A.M.	1	0	0.4308	0.237	0.00087	1.49	0.0013	0.4321	5.0049	4.741			
" 23..	8.52 "	5	2	0.6215	0.226	0.00079	2.17	0.0017	0.6232	3.0070	4.736	2.8958	0.089	
" 24...	9.00 "	6	2	0.7478	0.219	0.00075	2.21	0.0016	0.7494	2.3125	4.736	2.1984	0.076	
" 25...	9 08 "	4	1	0.5764	0.229	0.00081	2.02	0.0016	0.5780	3.3104	4.736	3.2405	0.121	
" 26...	9.47 "	4	1	0.5766	0.229	0.00081	2.02	0.0016	0.5782	3.3104	4.736	3.2387	0.124	
" 27...	9.54 "	1	0	0.4302	0.237	0.00087	1.49	0.0013	0.4315	5.0049	4.731			
Series VI.	March 22.													
Exp. 28...	10 04 A.M.	1	0	0.5115	0.301	0.00141	1.48	0.0021	0.5136	5.0046	6.132			
" 29..	10.13 "	2	1	0.5997	0.295	0.00135	1.82	0.0025	0.6022	4.0063	6.134	3.9475	0.098	
" 30...	10.18 "	3	1	0.5996	0.295	0.00135	1.82	0.0025	0.6021	4.0050	6.134	3.9485	0.094	
" 31...	10.25 "	5	2	0.7398	0.285	0.00126	2.16	0.0027	0.7425	3.0070	6.134	2.8869	0.081	
" 32 ..	10 31 "	4	1	0.6860	0.289	0.00130	2.01	0.0026	0.6886	3.3101	6.134	3.2311	0.115	
" 33...	10 38 "	6	2	0.8905	0.275	0.00118	2.19	0.0026	0.8931	2.3125	6.134	2.1902	0.068	
" 34...	10.46 "	1	0	0.5117	0.301	0.00141	1.48	0.0021	0.5138	5.0047	6.136			
Series VII	March 22.													
Exp. 35...	11.02 A.M.	6	2	0.9548	0.301	0.00141	2.18	0.0031	0.9579	2.3126	6.796	2.1851	0.067	
" 36...	11.14 "	1	0	0.5477	0.331	0.0017	1.48	0.0025	0.5502	5.0046	6.796			
Ser. VIII.	March 21.													
Exp. 37...	9.36 A.M.	1	0	0.6010	0.375	0.00219	1.47	0.0032	0.6042	5.0040	7.814			
" 38...	9.45 "	2	1	0.7055	0.366	0.00208	1.81	0.0038	0.7093	4.0076	7.815	3.9384	0.098	
" 39...	9.50 "	3	1	0.7064	0.366	0.00208	1.81	0.0038	0.7102	4.0064	7.815	3.9309	0.106	
" 40...	9.56 "	5	2	0.8714	0.352	0.00193	2.14	0.0041	0.8755	3.0104	7.815	2.8747	0.077	
" 41...	10.03 "	1	0	0.6011	0.375	0.00219	1.47	0.0032	0.6043	5.0040	7.816			
" 42...	10.11 "	4	1	0.8062	0.357	0.00198	2.00	0.0040	0.8102	3.3112	7.815	3.2281	0.103	
" 43..	11.05 "	4	1	0.8063	0.357	0.00198	2.00	0.0040	0.8103	3.3110	7.804	3.2231	0.108	
" 44...	11.12 "	1	0	0.6002	0.375	0.00219	1.47	0.0032	0.6034	5.0039	7.798			
" 45...	11.20 "	5	2	0.8702	0.352	0.00193	2.14	0.0041	0.8743	3.0098	7.800	2.8750	0.077	
Series IX.	March 22.													
Exp. 46...	2.56 P.M.	1	0	0.6925	0.455	0.00322	1.46	0.0047	0.6972	5.0042	9.678			
" 47...	3.23 "	1	0	0.6924	0.455	0.00322	1.46	0.0047	0.6971	5.0042	9.676			
" 48...	3.30 "	4	1	0.9317	0.430	0.00287	1.99	0.0057	0.9374	3.3095	9.667	3.2105	0.106	
" 49...	4.07 "	4	1	0.9297	0.428	0.00285	1.99	0.0057	0.9354	3.3095	9.630	3.2084	0.108	
" 50...	4.16 "	1	0	0.6897	0.452	0.00318	1.46	0.0046	0.6943	5.0042	9.618			
Series X.	March 22.													
Exp. 51...	11.25 A.M.	2	1	0.9448	0.539	0.00452	1.79	0.0081	0.9529	4.0065	12.147	3.9363	0.074	
" 52...	11.33 "	3	1	0.9432	0.539	0.00452	1.79	0.0081	0.9513	4.0043	12.147	3.9461	0.061	
" 53...	11.42 "	1	0	0.8047	0.556	0.00480	1.45	0.0070	0.8117	5.0038	12.147			
" 54...	11 50 "	2	1	0.9460	0.539	0.00452	1.79	0.0081	0.9541	4.0065	12.147	3.9288	0.081	

The absolute values of the co-efficient b were increased in all cases; from 20 to 35 per cent. with the smaller depths on the weir, and from 3 to 5 per cent. with the largest depths.

The marked decrease in the co-efficients with the increasing depths on the weir was still more noticeable than before. The variation in the co-efficients, occasioned by the different forms of weir used, remained practically unchanged.

The comparisons just given indicate that there is no probable error in the formulæ for discharge which would be sufficient to account for the marked variation found in the co-efficient.

The correction for velocity of approach, as given by us, is much larger than that generally used; but the effect of reducing this correction would be to *increase* the peculiar variations of the co-efficients.

One other feature of the experiments may be noticed. In the case of the weir without end contractions, there was some friction against the boards forming the ends of the weir, and in consequence the effect of end contraction, as obtained from the experiments, is not its total effect; but it is diminished by a part or the whole of the end friction. This may have some effect upon the absolute value of the co-efficients; but it does not seem probable that it would change much their relations to each other.

With regard to the practical application of this series of experiments we can say but little.

The different forms of weir have very different co-efficients, and the variety of forms experimented upon is not sufficient for determining the laws by which the co-efficients vary. Within the limits of the experiments the results at each form of weir are represented fairly by the straight lines shown on the diagram (Plate XI); but it is evident that these lines could not be extended to large depths, else the depth would be reached at which the correction would be zero, which is absurd.

With depths on the weir smaller than three inches, it is probable that the cohesion of the water and its adhesion to the ends of the weir would diminish the effect of the end contraction. For the reasons just given, we would limit the application of these experiments to cases in which the form of and depths on the weir are the same as at those experimented upon.

The chief value of these experiments consists in the warning they give that this form of weir should be avoided in cases where the most

accurate results are desired. In this connection it should be remembered that for weirs with end contractions, the effect of velocity of approach varied largely with the form of the weir. (See page 21 *et seq.*)

When it is necessary to change the length of the weir to accommodate varying volumes of water, the introduction of end contraction is a great convenience. In this case we would recommend the use of one end contraction rather than two.

RECAPITULATION.

The following statements review in a summary manner the different formulæ given, and the principal limits to their applicability. It may also serve as an index to the portions of the paper which treat of the practical application of the formulæ.

The general formula for the discharge over the simplest form of sharp-crested weir is

$$Q = 3.31\, Lh^{\frac{3}{2}} + 0.007\, L.$$ (See page 82.)

To allow the use of this formula without any correction, the weir should have a level crest and vertical ends; it should be made in a dam vertical on its up-stream side; end contractions should be suppressed; the width of the crest should be so narrow that the water will not touch it after passing the up-stream edge, which should be sharp; the depth on the weir should be measured from the surface of the water above the curvature of the sheet; air should have free access to the space beneath the sheet of water, and, when this condition is fulfilled, the effect of raising the water on the down-stream side to the level of the crest will not be noticeable, provided the water has considerable depth; nor will the error exceed one per cent. if the water is allowed to rise above the level of the crest until 15 per cent. of the depth on the weir is submerged. (See page 103.)

The formula does not apply to depths on the weir less than 0.07 feet. When the depths become large its application will be limited by the difficulty in making the proper correction for velocity of approach. This last condition cannot be entirely eliminated in any case, though its effect, in some cases, may be neglected; when it becomes important, correction will be made for its effect by adding to the observed depth on the weir; to make this correction accurately, it is necessary that the

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FTELEY AND STEARNS
ON FLOW OF WATER

PLATE XI.

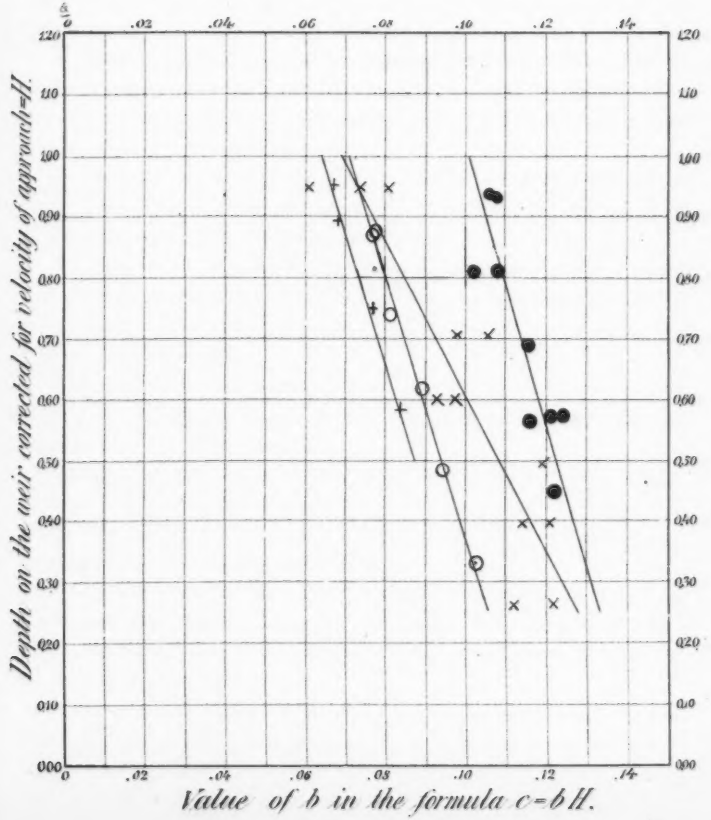
DIAGRAM SHOWING THE RESULTS OF EXPERIMENTS UPON THE EFFECT OF END CONTRACTION.

EXPLANATIONS.

Experiments upon weirs Nos. 2 & 3 are shown thus :— ×
" " " " 4 " " " ●
" " " " 5 " " " ○
" " " " 6 " " " +

The description of the different weirs is given in table XXVII.

The lines are intended to show the mean results of the experiments upon each form of weir.



channel of approach should be unobstructed for a sufficient distance from the weir, and rectangular and uniform in section; also that the depth on the weir should be taken in a proper manner, and that the velocities in different parts of the cross-section be nearly uniform.

If sufficient care is taken the correction can be made without important error when the depths of the channel below the crest is only 0.5 feet; it is better, however, to make the channel as deep as possible.

The correction for velocity of approach is furnished by the formula

$$C = ch.$$ (See page 10).

In the case of the weir without end contraction c has a general value of about 1.5; but can be obtained more accurately for different circumstances from Table I., page 12. In some instances, labor may be saved by taking the correction directly from the diagram on Plate V., page 52.

When end contraction exists c has a general value of 2.05, but a somewhat more accurate correction may be obtained by following the rules given on page 21. The correction for this form of weir is less definite than in the preceding case.

If the head is taken at the bottom of the channel near the weir, a correction can be made for it (see pages 24–30); but it is better, when practicable, not to take the head at that point.

The conditions required for the proper application of the general weir formula can usually be complied with except in the case of velocity of approach. If necessary, however, a correction can be made for the following modifications:

End Contraction.—Correction for the effect of end contraction can be made by reducing the length of the weir, using for this purpose the formula $C = bH$. (See page 109.)

The general value of b may be called 0.1, as recommended by Mr. Francis. Our own experiments show, however, that the value of b varies largely with the form of and depth on the weir, so that any constant co-efficient is but an approximation.

The co-efficients for the forms of weir experimented upon may be taken from the diagram, Plate XI., page 112. With the present information on the subject, end contraction is a source of error, and should be avoided if practicable.

Width of Crest.—When the depth on the weir is more than 1.6 times the width of the crest, it is necessary to observe the sheet and notice

whether it adheres to the crest or lifts entirely from it, touching only the up-stream edge.

In the latter case it is obvious that no correction is necessary.

In all cases in which the sheet adheres to the crest, modify the observed depth on the weir by adding, algebraically, the corrections given in Table XXIII., page 96. For the limits to the applicability of this formula see page 95.

Up-stream Edge of the Weir Rounded.—If the up-stream edge of the weir is a small quarter circle, add seven-tenths of its radius to the depth on the weir before applying the general weir formula. (See page 98.)

Height of Water on the Down-stream Side.—When the water on the down-stream side rises above the level of the crest, use the formula for a submerged weir,

$$Q = c\,l\left(d + \frac{d'}{2}\right)\sqrt{h}. \quad \text{(See page 102.)}$$

The value of c in this formula varies with the ratio of $\frac{d'}{d}$ and may be taken from Table XXVI., page 105. For the limits to the applicability of the formula, see page 106.

FLOW IN THE CONDUIT.

As it has been mentioned in the beginning of this paper, the weirs described in these pages have been used for ascertaining the flowing capacity of the Sudbury River Conduit. The experiments made for this purpose were conducted with care, and, to do justice to them, they should be described at some length; but as this paper may be found already too long, and because circumstances might prevent the preparation of another with sufficient details, it may be well to indicate briefly here the results obtained.

The conduit is generally nine feet wide and seven feet eight inches high, of the form shown by Fig. 3, Plate II. Its interior lining is of brick, built to conform very closely to the theoretical section; but, for the purposes of the experiments, numerous measurements were made to ascertain its exact size. The tunnels, siphon and other portions of the conduit on bridges or embankments vary from the standard section just described, either in the nature of the lining or in their form, or in both,

thus permitting experiments to be made upon the flow under different conditions.

The inclination of the conduit is one foot per mile.

The well-known formula of Chezy, $v = c\sqrt{RI}$, was selected to calculate the results, on account of its simple form.

In this formula

v represents the velocity in feet per second,
c " a co-efficient,
R " the hydraulic mean depth,
I " the sine of the inclination.

c was found to be a variable quantity, which increased with the hydraulic mean depth and with the smoothness of the lining.

Table XXIX., which has been prepared from the mean results of the principal series of experiments on the flow of the conduit where it is lined with brick, gives the various values of c corresponding to hydraulic mean depths varying from 0.1 feet to 2.7 feet.

TABLE XXIX.

VALUES OF THE CO-EFFICIENT c IN THE FORMULA $v = c\sqrt{RI}$.

The Limits of the Experiments are from $R = 0.5$ to $R = 2.33$ feet.

Hydraulic Mean Radius R.	Co-efficient c.	Hydraulic Mean Radius R.	Co-efficient c.	Hydraulic Mean Radius R.	Co-efficient c.
0.1	96.3	1.0	127.0	1.9	137.1
0.2	104.7	1.1	128.5	2.0	137.8
0.3	109.9	1.2	129.8	2.1	138.5
0.4	113.8	1.3	131.1	2.2	139.1
0.5	116.9	1.4	132.2	2.3	139.6
0.6	119.4	1.5	133.3	2.4	140.0
0.7	121.7	1.6	134.4	2.5	140.4
0.8	123.6	1.7	135.3	2.6	140.6
0.9	125.4	1.8	136.2	2.7	140.8

The formula

$$v = 127\, R^{0.62}\, I^{0.5},$$

in which the letters v, R, I, have the same values as above, gives results identical with Table XXIX. when the value of R is below 1.6 feet. When R attains 2.5 feet it gives results 1 per cent. too large.

As the flowing capacity of a channel varies sensibly with slight changes in the nature of its lining and with its cleanliness, the condition of the conduit at the time of the experiments should be indicated.

The conduit had been built three years before of hard-burned bricks, with a cement joint one-quarter of an inch in thickness. Being below the level of the water table in the surrounding ground, portions of its sides and bottom were covered with a white coating which, for the purpose of the experiments, was removed as well as could be done with hoes and brooms; but the hardest portion of that deposit was left adhering to the brick-work.

Table XXX., which has been prepared for practical use in measuring the flow through the conduit, may have some general interest, and is given below:

TABLE XXX.

TABLE OF VOLUMES FLOWING IN THE CONDUIT IN MILLION GALLONS PER 24 HOURS, CORRESPONDING TO DEPTHS ABOVE CENTRE OF INVERT.

Ft.	0	.1	.2	.3	.4	.5	.6	.7	.8	.9
0	0	0.14	0.32	0.54	0.90	1.35	1.99	2.85	3.82	4.89
1.	6.04	7.29	8.61	9.99	11.41	12.87	14.37	15.93	17.55	19.23
2.	20.97	22.75	24.56	26.39	28.25	30.15	32.08	34.05	36.04	38.01
3.	40.00	42.02	44.08	46.16	48.25	50.34	52.44	54.52	56.62	58.71
4.	60.80	62.89	64.99	67.08	69.14	71.17	73.18	75.18	77.16	79.13
5.	81.06	82.95	84.80	86.62	88.43	90.20	91.92	93.58	95.15	96.65
6.	98.19	99.66	101.03	102.29	103.46	104.55	105.56	106.47	107.26	107.91
7.	108.41	108.79	108.99	108.99						

This table is based on weir measurements of quantities up to seventy-two million gallons per 24 hours; the rest of the table is calculated.

A portion of the conduit, six hundred feet in length, chosen on account of its uniformity of section and grade, was used in connection

with the smaller weir; the experiments were made six months after the conduit was built, when the white coating on the sides was soft and could be scraped off so as to leave a clean brick surface; in this case the flow was increased about 1½ per cent., and is represented by the formula

$$v = 129\, R^{0.62}\, I^{0.5}$$

Where the inside of the conduit is lined with a coating of mortar made of pure Portland cement, its flowing capacity is from 7 to 8 per cent. greater. This coating, although applied with floats, did not present as smooth a surface as was obtained in other portions of the conduit, where experiments would probably have given higher results.

In some parts of the conduit where the brick surface was covered with a wash of Portland cement laid with a brush, the flowing capacity was increased to the extent of from one to three per cent.

For a long tunnel 4 614 feet in length, in which the rock sides have been left ragged for 4 362 feet without any lining, but with a smooth concrete floor, the co-efficient in the same formula was found about 40 per cent. smaller than for the brick conduit, the hydraulic mean depth being the same.

Although the rock excavation is at no place less than two feet wider than the internal width of the brick conduit, a greater inclination of the water surface was required to flow the same volume. The depth of water in the tunnel during the tests was 3.44 feet.

For some iron, coated, siphon pipes, four feet in diameter, laid as a portion of the conduit, the co-efficient was found to be 143 against 127 for the same hydraulic mean depth in the brick conduit. 143 is the average of four co-efficients varying from 140.14 to 146.67, determined in as many experiments made with velocities from 2.616 feet to 6.195 feet.

Current Meter.

In connection with the construction of the works, the flow in Sudbury River and in other channels was measured frequently with current meters which were found very convenient, and, so far as could be determined by practical tests, very reliable. While observations were taken in the conduit to ascertain its flow, these instruments were accurately tested by comparing the velocities of the water, as indicated by them, with the correct velocities calculated from the weir measurements.

The current meter which was the most used during these observations had a measuring wheel 0.3 feet in diameter; the instrument was so protected as to permit the measurement of velocities within a very small distance of the sides and bottom of the conduit, and its construction was sufficiently delicate to allow of the registering of as low a velocity as 0.1 feet per second.

In order to ascertain the mean velocity of a water section, it was divided into a number of small squares equal in area, and the current meter was used in two ways, viz.:

First, by leaving it an equal length of time in the centre of each square, recording the velocity in each case and averaging the results.

Second, by moving it slowly with uniform velocity, by means of a special apparatus, throughout the whole section, in such a manner as to leave the measuring wheel an equal length of time in each square, and deducing the mean velocity from a single observation.

The meter had been originally rated in the usual way by moving it in a straight line with uniform velocities through still water; but as the water during that operation does not act on the instrument in exactly the same manner as when it is held still or moved transversely in a current, the accuracy of its readings was further tested by trying it under various conditions, and it was found that, whether the instrument is held still or moved continuously through the water (provided in this case the velocity of the motion does not exceed 5 per cent. of the velocity of the current); whether the velocity of the water is large or small; whether the current is regular or very irregular (as it was during some experiments when the flow was partially obstructed a short distance above the point of observation), the velocities recorded by the current meter did not vary more than one per cent. from those indicated by the weir measurement.

If the transverse motion of the meter is too rapid it will furnish too small results; an extreme case, where the rate of motion of the current meter was about two-thirds of the velocity of the water, showing an error of about 9 per cent.